

CONGRUENCY AND SIMILARITY – IGCSE MATHS

NAME SOLUTIONS = 40 marks

Question 1

Here are three similar quadrilaterals.

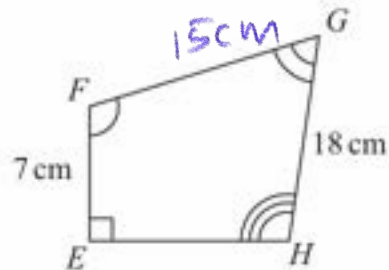
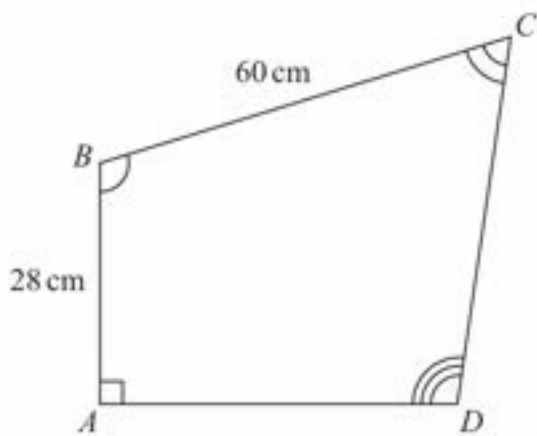
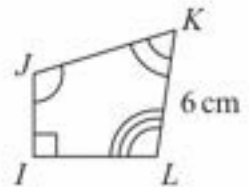


Diagram NOT accurately drawn



Work out the length of JK

$$\frac{7}{28} = \frac{FE}{60}$$

$$\frac{1}{4} = \frac{FE}{60}$$

$$FE = 15 \text{ cm}$$

$$\frac{6}{18} = \frac{JK}{15}$$

$$\frac{1}{3} = \frac{JK}{15}$$

$$JK = 5 \text{ cm}$$

5 cm

(3)

Question 2

Shape P is similar to shape Q

The table shows some information about shape P and shape Q

	Surface area (cm ²)	Volume (cm ³)
Shape P	200	672
Shape Q	450	

Work out the volume of shape Q

$$\begin{aligned}\text{Area Factor} &= \frac{450}{200} \\ &= \frac{9}{4}\end{aligned}$$

$$\text{Scale factor} = \frac{3}{2} \checkmark$$

$$\text{Volume F} = \frac{27}{8} \checkmark$$

$$\begin{aligned}\text{Volume of Q} &= \frac{27}{8} \times 672 \\ &= 2268 \text{ cm}^3 \checkmark\end{aligned}$$

cm³ [3]

Question 3

A and B are two similar solids.

The height of solid A is 31 cm

The height of solid B is 18.6 cm

Given that

$$\text{volume of solid A} - \text{volume of solid B} = 735 \text{ cm}^3$$

find the volume of solid A

$$\text{Scale factor} = \frac{31}{18.6} = \frac{5}{3}$$

$$\text{Volume F} = \frac{125}{27} \checkmark$$

$$125 - 27 = 98$$

$$1 \text{ part vol} = 735 \div 98 = 7.5 \checkmark$$

$$\begin{aligned}\text{Volume of A} &= 125 \times 7.5 \checkmark \\ &= \underline{937.5 \text{ cm}^3} \checkmark\end{aligned}$$

[4]

Question 4

The diagram shows two similar quadrilaterals, $ABCD$ and $EFGH$

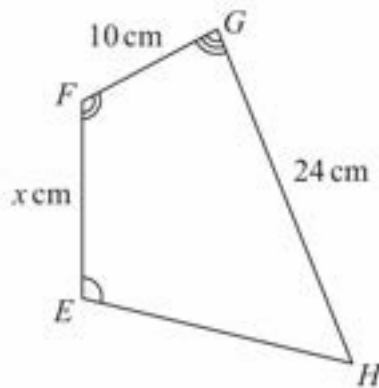
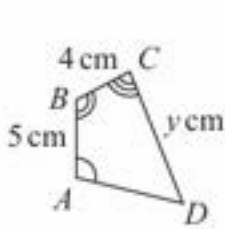


Diagram **NOT**
accurately drawn

$$AB = 5 \text{ cm} \quad BC = 4 \text{ cm} \quad CD = y \text{ cm}$$

$$EF = x \text{ cm} \quad FG = 10 \text{ cm} \quad GH = 24 \text{ cm}$$

(a) Work out the value of x

$$\frac{x}{5} = \frac{10}{4}$$

$$\frac{x}{5} = \frac{5}{2}$$

$$2x = 25$$

$$x = 12.5 \text{ cm}$$

$$x = \underline{\hspace{2cm}} \quad (2)$$

(b) Work out the value of y

$$\frac{y}{24} = \frac{4}{10}$$

$$\frac{y}{24} = \frac{2}{5}$$

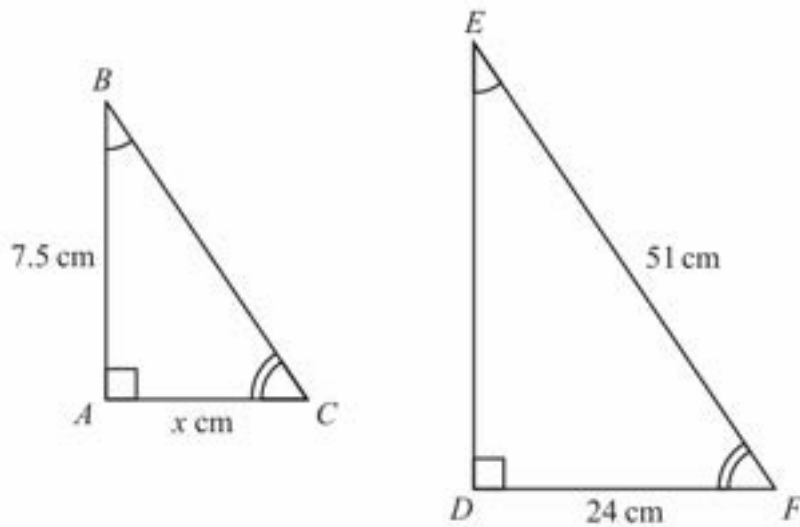
$$5y = 48$$

$$y = 9.6 \text{ cm}$$

$$y = \underline{\hspace{2cm}} \quad (2)$$

Question 5

Here are two similar triangles.



Diagrams **NOT** accurately drawn

Work out the value of x
Show your working clearly

$$ED = \sqrt{51^2 - 24^2} = 45 \text{ cm} \checkmark$$

$$\frac{x}{24} = \frac{7.5}{45} \checkmark$$

$$\frac{x}{24} = \frac{1}{6} \checkmark$$

$$6x = 24 \checkmark$$

$$x = 4 \text{ cm} \checkmark$$

.....[5]

Question 6

Solid A is similar to solid B

Here is some information about solid A and solid B

	solid A	solid B
Height (cm)	3^x	
Area (cm ²)	7776	486
Volume (cm ³)	8^x	2^{x+4}

Work out the height of solid B
Give your answer as a decimal.

$$\text{Area } F = \frac{486}{7776}$$

$$= \frac{1}{16}$$

$$\text{Scale } F = \frac{1}{4}$$

$$\text{Volume } F = \frac{1}{64}$$

$$\frac{1}{64} = \frac{2^{x+4}}{2^{3x}}$$

$$2^{3x} = 2^6 \times 2^{x+4}$$

$$2^{3x} = 2^{x+10}$$

$$2x = 10$$

$$x = 5$$

$$\frac{1}{4} = \frac{y}{243}$$

$$4y = 243$$

$$y = 60.75 \text{ cm}$$

Question 7

Here is a shape formed from two triangles ABC and CDE .
 ACD and BCE are straight lines.

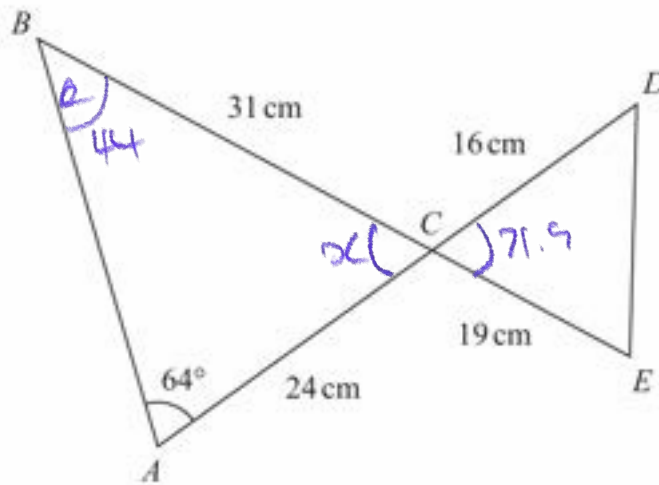


Diagram **NOT**
accurately drawn

$AC = 24 \text{ cm}$ $BC = 31 \text{ cm}$ $CE = 19 \text{ cm}$ $CD = 16 \text{ cm}$

Angle $BAC = 64^\circ$

Work out the length of DE

Give your answer correct to 3 significant figures.

$$\frac{24}{\sin \theta} = \frac{31}{\sin 64^\circ}$$

$$\sin \theta = \frac{24 \times \sin 64^\circ}{31}$$

$$\theta = 44.094\dots$$

$$\begin{aligned} x &= 180 - 64 - 44.094\dots \\ &= 71.90576\dots \end{aligned}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$DE = \sqrt{16^2 + 19^2 - 2 \times 16 \times 19 \times \cos 71.9^\circ}$$

$$= \underline{\underline{20.7 \text{ cm}}}$$

Question 8

A and B are two similar vases.



Diagram NOT
accurately drawn

$$64B = 25A$$

$$\frac{B}{A} = \frac{25}{64}$$

The vases are such that

$$\text{surface area of vase B} = \frac{25}{64} \times \text{surface area of vase A}$$

B A

Area : F = 25 64

and that

$$\text{volume of vase A} - \text{volume of vase B} = 541.8 \text{ cm}^3$$

Calculate the volume of vase B

$$\text{Scale factor} = \frac{5}{8} \quad \begin{matrix} \text{(B)} \\ \text{(A)} \end{matrix} \quad \checkmark$$

$$\text{Volume F} = \frac{125}{512} \quad \begin{matrix} \text{(B)} \\ \text{(A)} \end{matrix}$$

$$\begin{aligned} \text{Difference} &= 512 - 125 \\ &= 387 \end{aligned} \quad \checkmark$$

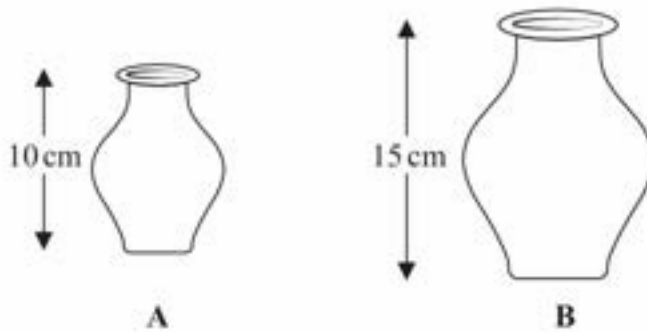
$$\begin{aligned} 1 \text{ part} &= \frac{541.8}{387} \\ &= 1.4 \end{aligned} \quad \checkmark$$

$$\text{Volume, B} = 1.4 \times 125$$

$$\underline{\underline{175}} \text{ cm}^3 \quad \checkmark \quad \mathbf{[4]}$$

Question 9

A and B are two similar vases.



Vase A has height 10 cm.

Vase B has height 15 cm.

The difference between the volume of vase A and the volume of vase B is 1197 cm^3

Calculate the volume of vase A

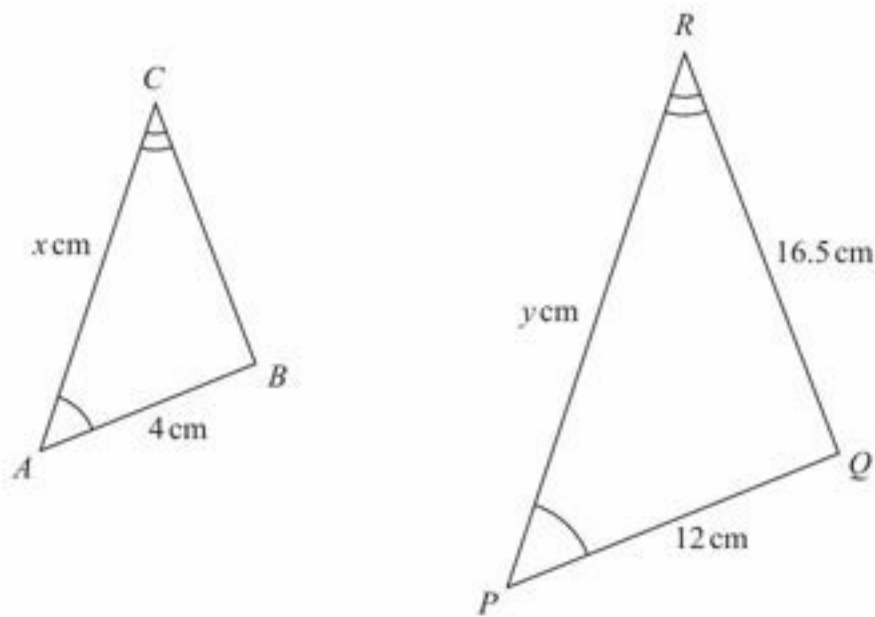
$$\begin{aligned} \text{Scale } F &= \frac{10}{15} \\ &= \frac{2}{3} \\ \text{Volume } F &= \frac{8}{27} \checkmark \end{aligned}$$

$$\begin{aligned} \rightarrow 1 \text{ part} &= \frac{1197}{19} \\ &= 63 \checkmark \\ \text{Vol A} &= 63 \times 8 \\ &= \underline{\underline{504 \text{ cm}^3}} \checkmark \end{aligned}$$

$$\begin{aligned} \text{Difference} &= 27 - 8 \\ &= 19 \checkmark \end{aligned}$$

$$\underline{\underline{504}} \text{ cm}^3 \quad \mathbf{[4]}$$

Question 10



Triangle ABC is similar to triangle PQR

$AB = 4\text{ cm}$ $PQ = 12\text{ cm}$ $RQ = 16.5\text{ cm}$ $AC = x\text{ cm}$ $PR = y\text{ cm}$

(a) Calculate the length of BC

$$\frac{BC}{16.5} = \frac{4}{12}$$

$$\frac{BC}{16.5} = \frac{1}{3}$$

$$3BC = 16.5$$

$$BC = 5.5\text{ cm}$$

..... cm
(2)

(b) Write down an expression for y in terms of x

$$\frac{y}{x} = \frac{12}{4}$$

$$\frac{y}{x} = 3$$

$$y = 3x$$

$$y = 3x$$

(1)

