A LEVEL MATHEMATICS

STATISTICS YEAR 2 – PROBABILITY

NAME SOLUTIONS 73 marks

Formulae

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B \mid A)$$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A')P(A')}$$

For independent events A and B,

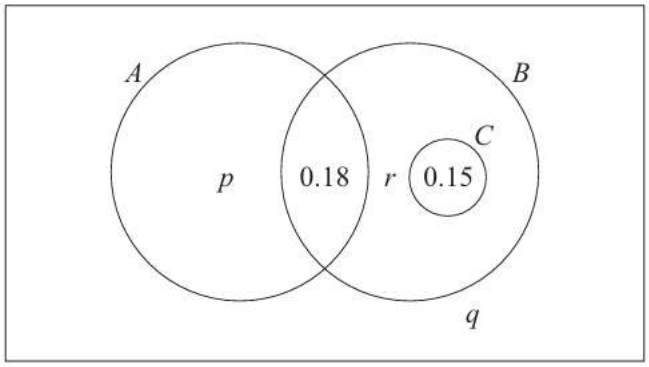
$$P(B \mid A) = P(B)$$
$$P(A \mid B) = P(A)$$
$$P(A \cap B) = P(A)\,P(B)$$

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Question 1

The Venn diagram, where p, q and r are probabilities, shows the events A, B and C and their associated probabilities.



$r = 0.27$
$p = 0.12$

(a) State a pair of mutually exclusive events from A, B and C

A and C (1)

Given that $P(A) = 0.3$

(b) find the value of p

$p + 0.18 = 0.3$
$p = 0.12$ (1) **(1)**

The events A and B are independent.

(c) Find $P(B)$

$P(A \cap B) = P(A) \times P(B)$ (1)
$0.18 = 0.3 \times P(B)$
$P(B) = 0.6$ (1) **(2)**

(d) Find $P(A' \cap B')$

$q = 1 - 0.72 = 0.28$ (1) **(1)**

(e) Find

(i) $P(B \mid C)$

(ii) $P(C \mid B)$

(3)

(i) $P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.15}{0.15} = 1$ (1)

(ii) $P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{0.15}{0.6} = \frac{1}{4}$ (1) (1)

Question 2

Bag **A** contains 5 red, 4 yellow and 3 green beads. = (12)

Bag **B** only contains red and yellow beads.

A bead is selected at random from bag **A** and a second bead is selected at random from bag **B**

Given that the probability that both beads selected are yellow is $\frac{3}{16}$

$\frac{4}{12} \times y = \frac{3}{16}$ (1)

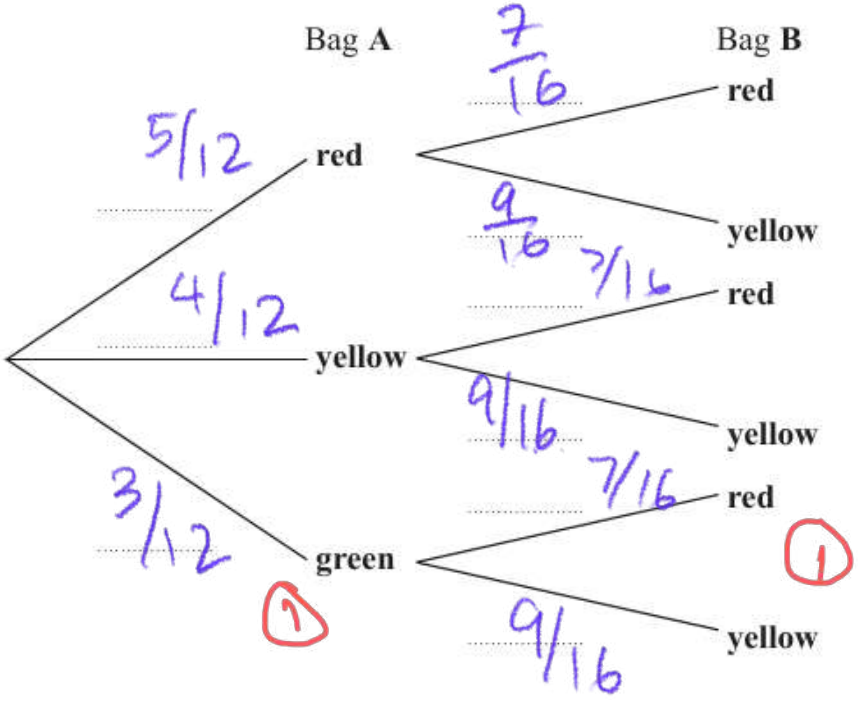
$y = \frac{3}{16} \times \frac{3}{1}$

(a) (i) find the probability of selecting a yellow bead from bag **B**

(2)

$y = \frac{9}{16}$ (1)

(ii) Hence complete the tree diagram below by finding the probability on each branch.



(1) (1)

(2)

(b) Find the exact probability that at least one of the two beads selected is yellow.

$1 - [P(GR + RR) = 1 - (\frac{3}{12} \times \frac{7}{16} + \frac{5}{12} \times \frac{7}{16}) = \frac{17}{24}$ (1) (1) (1) **(3)**

The event X is that at least one of the beads selected is yellow.

The event W is that a green bead is selected.

(c) Find the exact value of $P(W|X)$

(2)

$P(W|X) = \frac{P(W \cap X)}{P(X)}$

$= \frac{\frac{1}{4} \times \frac{9}{16}}{17/24}$ (1) $= \frac{27}{136}$ (1)

Question 3

Xian rolls a fair die 10 times.

The random variable X represents the number of times the die lands on a six. $X \sim B(10, \frac{1}{6})$

(a) Using a suitable distribution for X, find

Binomial
Tail (=, ≤, ≥, etc)
X value = 3
Trials = 10
$P = 1/6$

(i) $P(X = 3)$ $= 0.155045$ (1)

(ii) $P(X < 3)$ $= P(X \leq 2)$ (1)
$= 0.7752$ (1)

(3)

Xian repeats this experiment each day for 60 days and records the number of days when $X = 3$ $X \sim B(60, 0.155045)$ (1) (1)

(b) Find the probability that there were at least 12 days when $X = 3$

$P(X \geq 12) = 0.2118$ (1)

(3)

(c) Find an estimate for the total number of sixes that Xian will roll during these 60 days.

Total rolls $= 60 \times 10 = 600$, $E(X) = np = 600 \times \frac{1}{6} = 100$ (1)

(1)

(d) Use a normal approximation to estimate the probability that Xian rolls a total of more than 95 sixes during these 60 days.

$X \sim N(np, npq) \Rightarrow X \sim N(100, 83.3)$ (1)

(4)

Continuity correction: 95 sixes means 96 or more

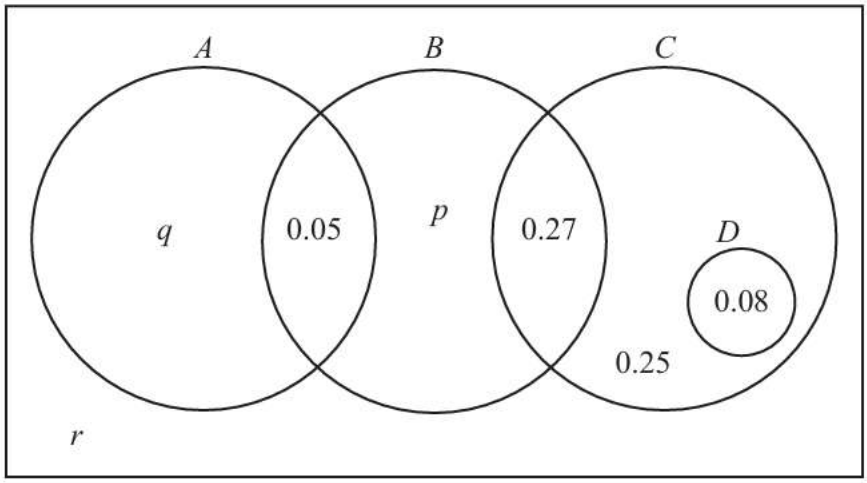
$\therefore$ use $P(X \geq 95.5)$ (1)

$= P(Z \geq -0.493)$ (1)

$= 0.689$ (1)

Question 4

The Venn diagram, where p, q and r are probabilities, shows the events A, B, C and D and associated probabilities.



(a) State any pair of mutually exclusive events from A, B, C and D A+C, A+D, A+C ①

(1)

$P(B \cap C) = P(B) \times P(C)$

The events B and C are independent. $0.27 = (p + 0.32) \times 0.6$ ①

(b) Find the value of p $p = 0.13$ ①

(2)

(c) Find the greatest possible value of $P(A \mid B')$

$r + q = 0.22$

$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.22}{0.55} = 0.4$

(3)

Given that $P(B \mid A') = 0.5$ $P(B) = 0.13 + 0.05 + 0.27 = 0.45$

(d) find the value of q and the value of r $P(B') = 0.55$ ①

(3)

$P(A \cap B') = q$

$\therefore P(A|B') = \frac{q}{0.55}$

$q + r + 0.78 = 1$

$q + r = 0.22$

Greatest value of q is when $r = 0$

$\therefore q = 0.22$ ①

$P(A|B') = \frac{0.22}{0.55} = 0.4$ ①

(e) Find $P([A \cup B]' \cap C)$ 0.33 ①

(1)

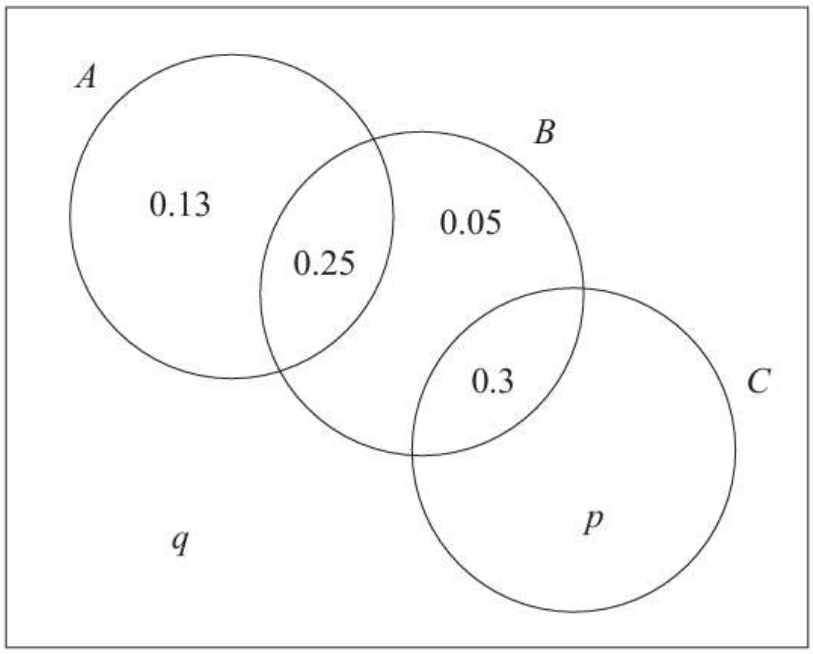
(f) Use set notation to write an expression for the event with probability p

(1)

$B \cap (A \cup C)'$ ①

Question 5

. The Venn diagram, where p and q are probabilities, shows the three events A, B and C and their associated probabilities.



(a) Find P(A)

$0.13 + 0.25 = 0.38$ (1)

(1)

The events B and C are independent.

(b) Find the value of p and the value of q

(3)

(c) Find P($A|B'$)

(2)

(b)

$P(B \cap C) = P(B) \times P(C)$

$0.3 = 0.6 \times (p + 0.3)$ (1)

$p = \frac{0.3}{0.6} - 0.3$

$p = 0.2$ (1)

$q = 0.07$ (1)

(c) $P(A|B') = \frac{P(A \cap B')}{P(B')}$

$= \frac{0.13}{0.13 + 0.2 + 0.07}$ (1)

$= 0.325$ (1)

Question 6

A company has 1825 employees.
The employees are classified as professional, skilled or elementary.

The following table shows

- the number of employees in each classification
- the two areas, *A* or *B*, where the employees live

	A	*B*	
Professional	740	380	1120
Skilled	275	90	365
Elementary	260	80	340
	1275	550	1825

An employee is chosen at random.

Find the probability that this employee

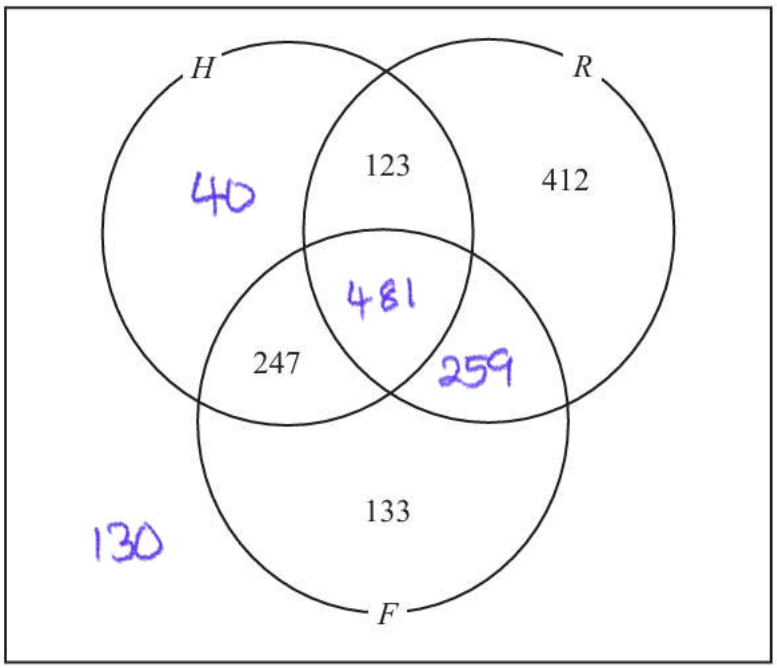
(a) is skilled, $\frac{365}{1825} = \frac{1}{5}$ **(1)**

(b) lives in area *B* and is not a professional. $\frac{170}{1825}$ **(1)**

Some classifications of employees are more likely to work from home.

- 65% of professional employees in both area *A* and area *B* work from home $= 728$
- 40% of skilled employees in both area *A* and area *B* work from home $= 146$
- 5% of elementary employees in both area *A* and area *B* work from home $= 17$
- Event *F* is that the employee is a professional
- Event *H* is that the employee works from home
- Event *R* is that the employee is from area *A*

(c) Using this information, complete the Venn diagram on the opposite page. **(4)**



(d) Find $P(R' \cap F)$ $\frac{380}{1825}$ **(1)**

(e) Find $P([H \cup R]')$ $\frac{263}{1825}$ **(1)**

(f) Find $P(F \mid H)$ $= \frac{P(F \cap H)}{P(H)}$ **(2)**

$= \frac{247 + 481}{891}$

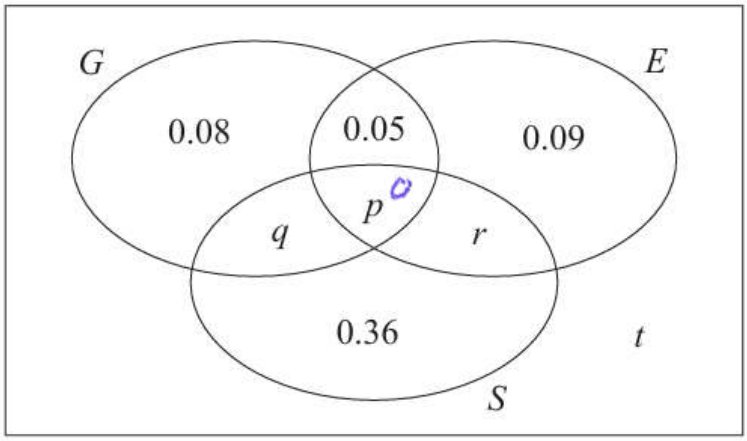
$= \frac{728}{891}$

Question 7

A large college produces three magazines.
One magazine is about green issues, one is about equality and one is about sports.
A student at the college is selected at random and the events G, E and S are defined as follows

G is the event that the student reads the magazine about green issues
E is the event that the student reads the magazine about equality
S is the event that the student reads the magazine about sports

The Venn diagram, where p, q, r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in the college who read exactly one of these magazines.

$0.08 + 0.36 + 0.09$

$= 0.53$ (1)

(1)

No students read all three magazines and $P(G) = 0.25$

(b) Find

(i) the value of p $p = 0$ (1)

(ii) the value of q

$0.08 + 0.05 + q = 0.25$ (1)

$q = 0.12$ (1)

(3)

Given that $P(S \mid E) = \frac{5}{12}$

(c) find

(i) the value of r

(ii) the value of t

(4)

(i) $\frac{P(S \cap E)}{P(E)} = \frac{5}{12}$

$\frac{r}{0.14+r} = \frac{5}{12}$ (1)

$12r = 0.7 + 5r$

$7r = 0.7$

$r = 0.1$ (1)

(ii) All probabilities $= 1$

$0.8 + t = 1$ (1)

$t = 0.2$ (1)

(d) Determine whether or not the events $(S \cap E')$ and G are independent.
Show your working clearly.

(3)

$(S \cap E') \cap G = P(S \cap E') \times P(G)$ (1)

$0.12 = (0.12 + 0.36) \times 0.08 + 0.05 + 0.12$

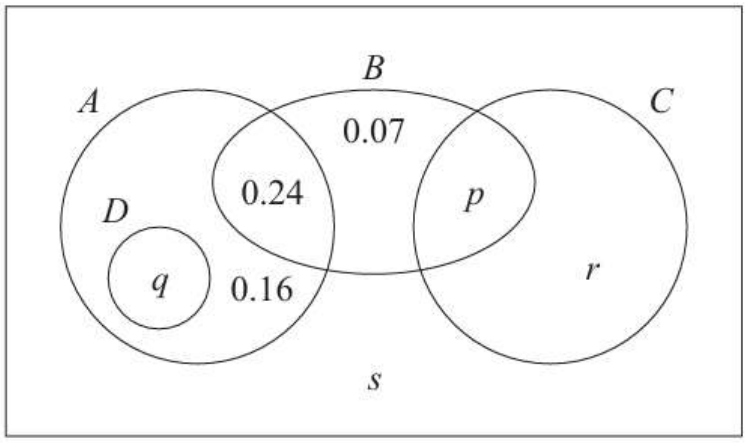
$0.12 \stackrel{?}{=} 0.48 \times 0.25$ (1)

$0.12 = 0.12$

Hence independent (1)

Question 8

The Venn diagram shows the probabilities associated with four events, A, B, C and D



(a) Write down any pair of mutually exclusive events from A, B, C and D

A + C, B + D, D + C (1)

(1)

Given that $P(B) = 0.4$

(b) find the value of p

$0.07 + 0.24 + p = 0.4$

$p = 0.09$ (1)

(1)

Given also that A and B are independent

(c) find the value of q

$P(A \cap B) = P(A) \times P(B)$

$0.24 = (0.4 + q) \times 0.4$ (1)

$0.6 = q + 0.4$ (1)

$q = 0.2$

(2)

Given further that $P(B' | C) = 0.64$

(d) find

$\frac{P(B' \cap C)}{P(C)} = 0.64$

(i) the value of r

(ii) the value of s

(4)

(i) $\frac{r}{r + 0.09} = 0.64$ (1)

$r = 0.64r + 0.0576$

$r = 0.16$ (1)

(ii) $\Sigma p = 1$

$0.92 + s = 1$ (1)

$s = 0.08$ (1)