AS MATHEMATICS

GRAPHS AND TRANSFORMATIONS

NAME Solutions 67 marks

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Question 1

(a) Sketch the curve with equation

$$y = \frac{k}{x} \qquad x \neq 0$$

where k is a positive constant.

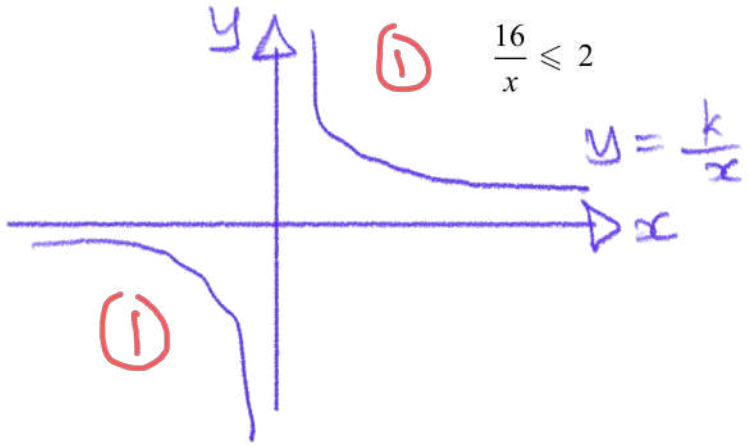
(2)

(b) Hence or otherwise, solve

$$\frac{16}{x} \leqslant 2$$

(3)

(a)



$\frac{16}{x} \leq 2$ (1)

$16 \leq 2x$

$8 \leq x$ (1)

$x < 0$, $x \geq 8$ (1)

Question 2

In this question you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = 8 - 10x + 6x^2 - x^3$

The curve C_2 has equation $y = x^2 - 12x + 14$

(a) Verify that when $x = 1$ the curves C_1 and C_2 intersect.

(2)

The curves also intersect when $x = k$.

Given that $k < 0$

(b) use algebra to find the exact value of k.

(5)

(a) $x=1$: $y_1(1) = 8 - 10(1) + 6(1)^2 - (1)^2$

$= 3$

$y_2(1) = (1)^2 - 12(1) + 14 = 3$ (1)

Same y coordinate of 3

$\therefore$ intersect at $(1, 3)$ (1)

(b) $8 - 10k + 6k^2 - k^3 = k^2 - 12k + 14$

$\therefore k^3 - 5k^2 - 2k + 6 = 0$ (1)

$(k-1)(k^2 - 4k - 6) = 0$ (1)

$k = 1, \quad k^2 - 4k - 6 = 0$

$(k-2)^2 - 4 - 6 = 0$ (1)

$(k-2)^2 - 10 = 0$

$k = 2 \pm \sqrt{10}$ (1)

hence $k = 2 - \sqrt{10}$ (1)

Question 3

(a) Factorise completely $x^3 + 10x^2 + 25x$

$x(x^2 + 10x + 25)$ (1)

$= x(x+5)^2$ (1)

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

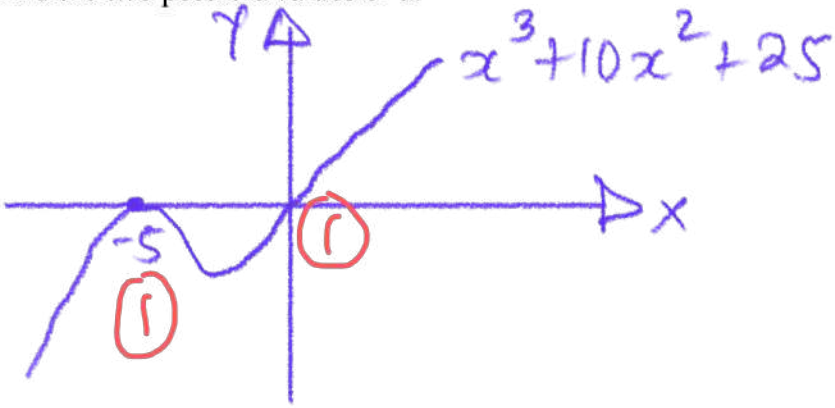
$$y = (x+a)^3 + 10(x+a)^2 + 25(x+a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

(b)



(c) $0 = (a-3)^3 + 10(a-3)^2 + 25(a-3)$

$(a-3)(a+2)^2 = 0$ (1)

$a = 3$ (1) or $a = -2$ (1) twice

Question 4

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

Sub $x = -2$: $4(-2)^3 - 12(-2)^2 - 15(-2) + 50 = 0$ (1)

$\therefore x+2$ is a factor (1)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

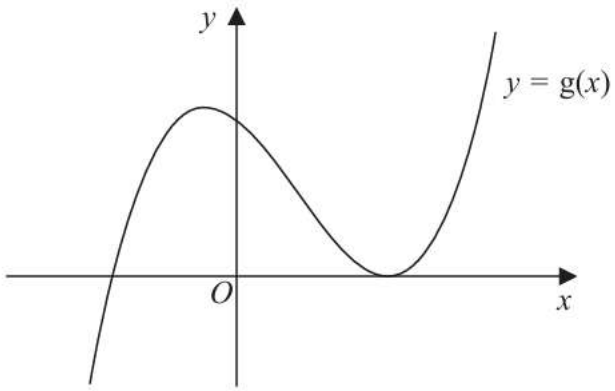


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

Sketch SC $\frac{1}{2}$ $\therefore x = -1, \quad x = 1.25$

(i) $x \leq -2, \quad x = 2.5$ (1)

(b)
$$\begin{array}{r} 4x^2 - 20x + 25 \\ x+2 \overline{)\,4x^3 - 12x^2 - 15x + 50} \\ -(4x^3 + 8x^2) \\ \hline -20x^2 - 15x + 50 \\ -(-20x^2 - 40x) \\ \hline 25x + 50 \\ 25x + 50 \\ \hline 0 \end{array}$$

(1) (1) (1)

$(x+2)(4x^2 - 20x + 25)$ (1)

$(x+2)(4x - 10)(4x - 10)$

$(x+2)(2x - 5)^2$

(ii) $g(2x) = 0$ (3)

$g(2x) = (2x+2)(4x-5)^2$

$= 0$

$2x + 2 = 0$

$x = -1$ (1)

$4x - 5 = 0$ (1)

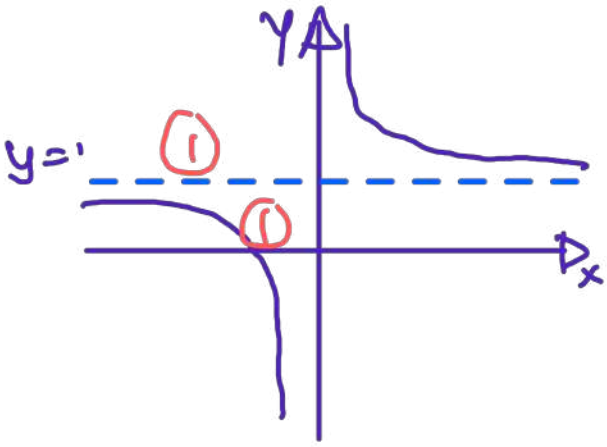
$x = 1.25$

Question 5

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.



(a) Sketch C stating the equation of the horizontal asymptote.

$y = 1$, $x = 0$ (1)

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

(b) $\frac{k^2}{x} + 1 = -2x + 5$

$k^2 + x = -2x^2 + 5x$ (1)

$2x^2 - 4x + k^2 = 0$ (1)

(c) $b^2 - 4ac = 0$

$(-4)^2 - 4(2)(k^2) = 0$ (1)

$16 - 8k^2 = 0$

$8k^2 = 16$ (1)

$k^2 = 2$

$k = \pm\sqrt{2}$ (1)

Question 6

(a) Factorise completely $9x - x^3$

$x(9 - x^2) = x(3-x)(3+x)$ (1) (1)

(2)

The curve C has equation

$$y = 9x - x^3$$

$x = 0, \quad x = -3, \quad x = 3$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

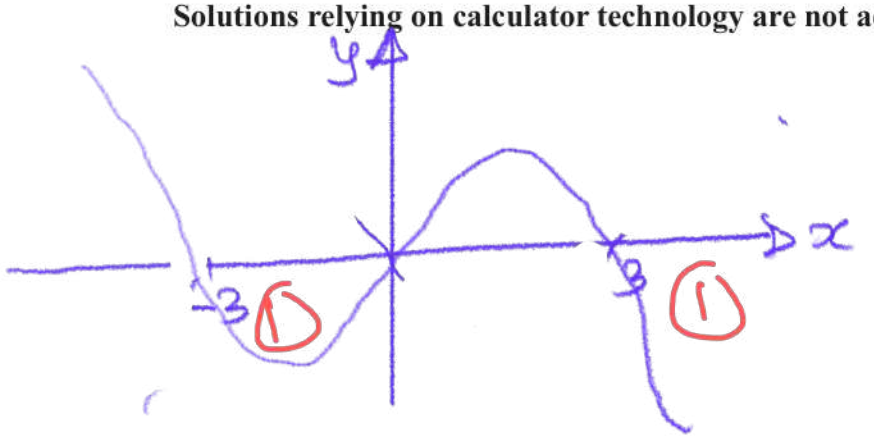
Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

(b)



(1) (1)

(c) at max / min point, $\frac{dy}{dx} = 0$

$y = 9x - x^3$

$\frac{dy}{dx} = 9 - 3x^2$

$3(3 - x^2) = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

When $x = \sqrt{3}$

$y = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}$

When $x = -\sqrt{3}$

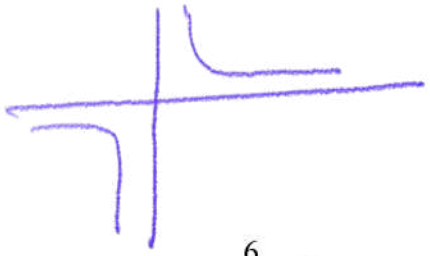
$-9\sqrt{3} + 3\sqrt{3} = -6\sqrt{3}$

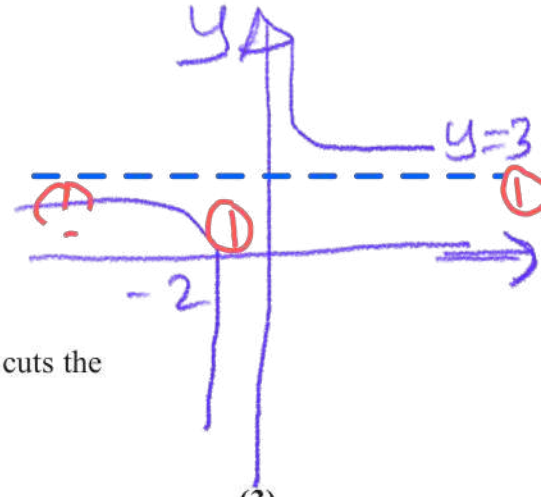
$-6\sqrt{3} < k < 6\sqrt{3}$

Question 7

The curve C_1 has equation

$$y = \frac{6}{x} + 3$$





(a) (i) Sketch C_1 stating the coordinates of any points where the curve cuts the coordinate axes.

(ii) State the equations of any asymptotes to the curve C_1

$x = 0, y = 3$ (1)

(3)

$x = 0$

The curve C_2 has equation

$$y = 3x^2 - 4x - 10$$

(b) Show that C_1 and C_2 intersect when

$$3x^3 - 4x^2 - 13x - 6 = 0$$

(2)

Given that the x coordinate of one of the points of intersection is $-\frac{2}{3}$

$x + \frac{2}{3}$

$3x + 2$

(c) use algebra to find the x coordinates of the other points of intersection between C_1 and C_2

(*Solutions relying on calculator technology are not acceptable.*)

(4)

(b) $3x^2 - 4x - 10 = \frac{6}{x} + 3$ (1)

$3x^3 - 4x^2 - 10x = 6 + 3x$

$3x^3 - 4x^2 - 13x - 6 = 0$ (1)

(c)

$$3x+2 \overline{)\,3x^3 - 4x^2 - 13x - 6}$$

$x^2 - 2x - 3$ (1)

$-(3x^3 + 2x^2$ (1)

$-6x^2 - 13x - 6$

$-(-6x^2 - 4x$

$-9x - 6$

$-9x - 6$

0 (1)

$(3x+2)(x^2 - 2x - 3) = (3x+2)(x-3)(x+1)$

$x = -\frac{2}{3}$

$x = 3$

$x = -1$ (1)

Question 8

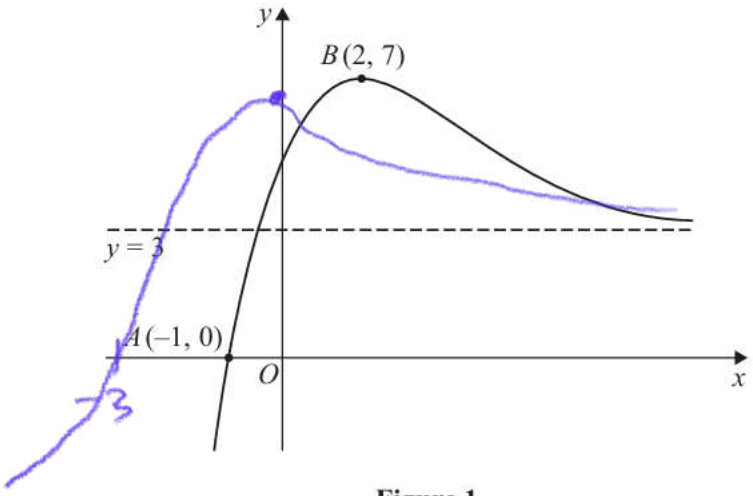


Figure 1

Figure 1 shows a curve with equation $y = f(x)$

The curve

- passes through the point $A(-1, 0)$
- has a maximum turning point at $B(2, 7)$
- has a horizontal asymptote with equation $y = 3$

On separate diagrams, sketch the curve with equation

(i) $y = f(x + 2)$

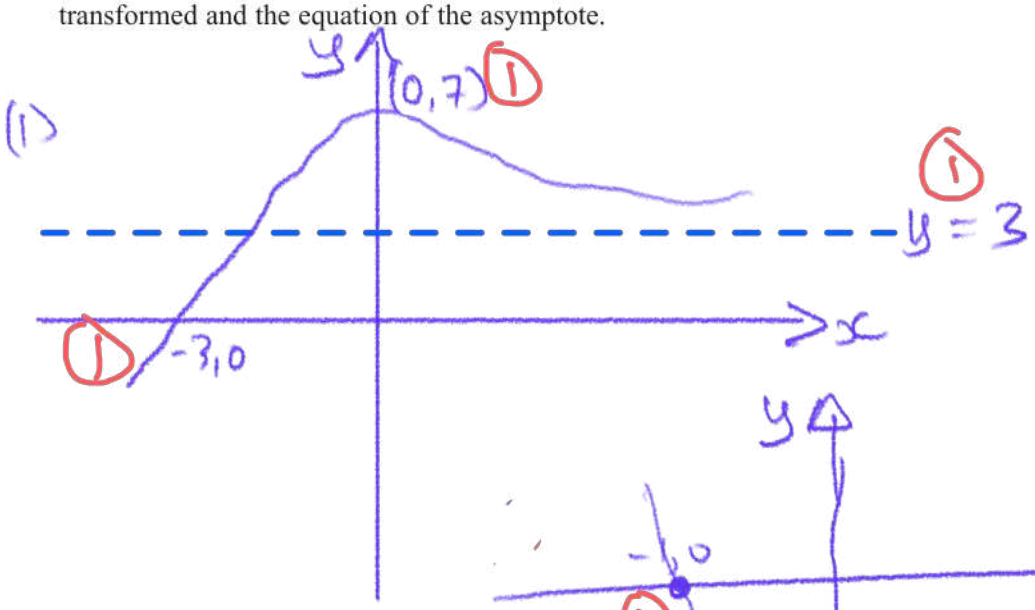
(3)

(ii) $y = -f(x)$

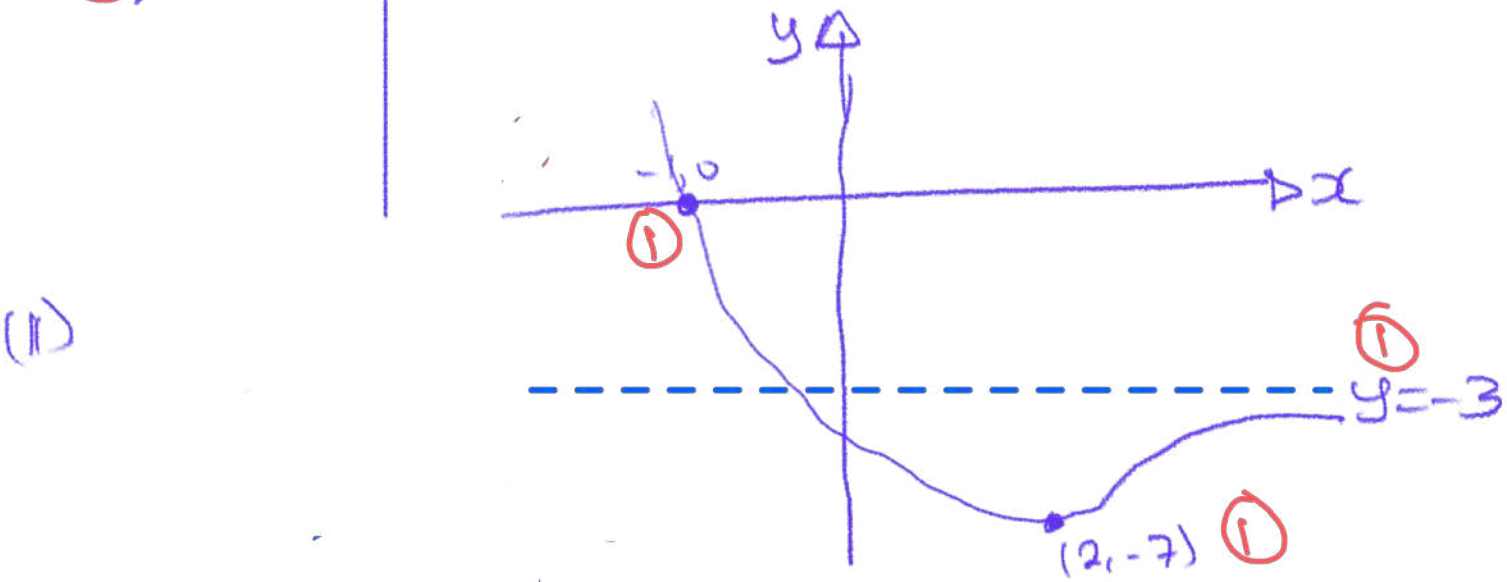
(3)

On each diagram, show clearly the coordinates of the points to which A and B are transformed and the equation of the asymptote.

(i)



(ii)



Question 9

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{4 - x} \qquad x \neq 4$$

On your sketch show

- the equation of the vertical asymptote
- the coordinates of the point of intersection of C with the y-axis

(3)

The straight line l has equation $y = 2x + k$, where k is a constant.

Given that l cuts C at least once,

(b) (i) show that

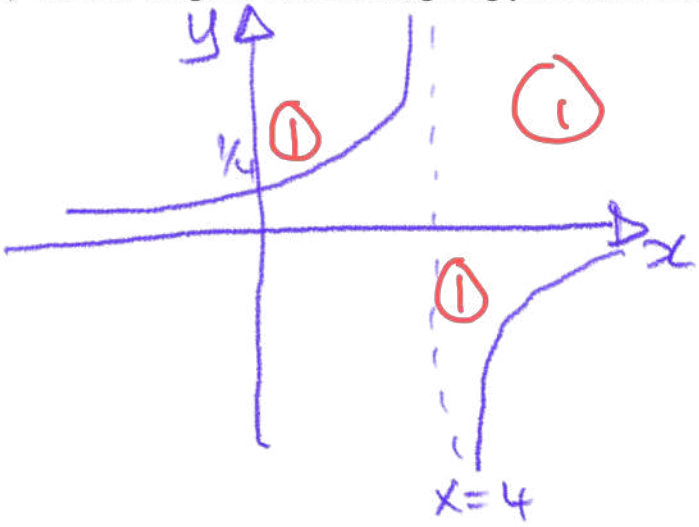
$$k^2 + 16k + 56 \geqslant 0$$

(4)

(ii) find the range of values of k, giving your answer in set notation.

(2)

a)



* horizontal asymptote $y=0$
* Vertical asympt. $x=4$
* y-intercept $(0, \frac{1}{4})$

b) $2x + k = \frac{1}{4-x}$ (1)

$(2x+k)(4-x) = 1$ (1)

$8x - 2x^2 + 4k - kx = 1$

$2x^2 + kx - 8x + 1 - 4k = 0$

$b^2 - 4ac \geqslant 0$

$(k-8)^2 - 4(2)(1-4k) \geqslant 0$ (1)

$k^2 - 16k + 64 - 8 + 32k \geqslant 0$

$k^2 + 16k + 56 \geqslant 0$ (1)

(ii) $\{k : k \leqslant -8 - 2\sqrt{2}\} \cup \{k : k \geqslant -8 + 2\sqrt{2}\}$ (1) (1)