

## AS STATISTICS PRACTICE 2

### Question 1

A magazine has a large number of subscribers who each pay a membership fee that is due on January 1st each year. Not all subscribers pay their fee by the due date. Based on correspondence from the subscribers, the editor of the magazine believes that 40% of subscribers wish to change the name of the magazine. Before making this change the editor decides to carry out a sample survey to obtain the opinions of the subscribers. He uses only those members who have paid their fee on time.

- (a) Define the population associated with the magazine.

All subscribers to the magazine

(1)

- (b) Suggest a suitable sampling frame for the survey.

A list of all paid up members

(1)

- (c) Identify the sampling units.

Paid up members

(1)

- (d) Give one advantage and one disadvantage that would have resulted from the editor using a census rather than a sample survey.

Adv: total accuracy

(2)

Disadv: time consuming to obtain data and analyse it.

As a pilot study the editor took a random sample of 25 subscribers.

- (e) Assuming that the editor's belief is correct, find the probability that exactly 10 of these subscribers agreed with changing the name.

$$X \sim B(25, 0.4)$$

$$P(X=10) = 0.1612$$

Use Binomial PD

$$X = 10$$

$$N = 25$$

$$p = 0.4$$

(3)

In fact only 6 subscribers agreed to the name being changed.

- (f) Stating your hypotheses clearly test, at the 5% level of significance, whether or not the percentage agreeing to the change is less than the editor believes.

(5)

$$H_0: p = 0.4$$

$$H_1: p < 0.4$$

$P(X \leq 6) = 0.0736 > 0.05$  hence accept  $H_0$   
and so percentage is less than the editor believes

## Question 2

In a study of how students use their mobile telephones, the phone usage of a random sample of 11 students was examined for a particular week.

The total length of calls,  $y$  minutes, for the 11 students were

17, 23, 35, 36, 51, 53, 54, 55, 60, 77, 110

- (a) Find the median and quartiles for these data.

(3)

Median = 53

$Q_1 = 35$

$Q_3 = 60$

A value that is greater than  $Q_3 + 1.5 \times (Q_3 - Q_1)$  or smaller than  $Q_1 - 1.5 \times (Q_3 - Q_1)$  is defined as an outlier.

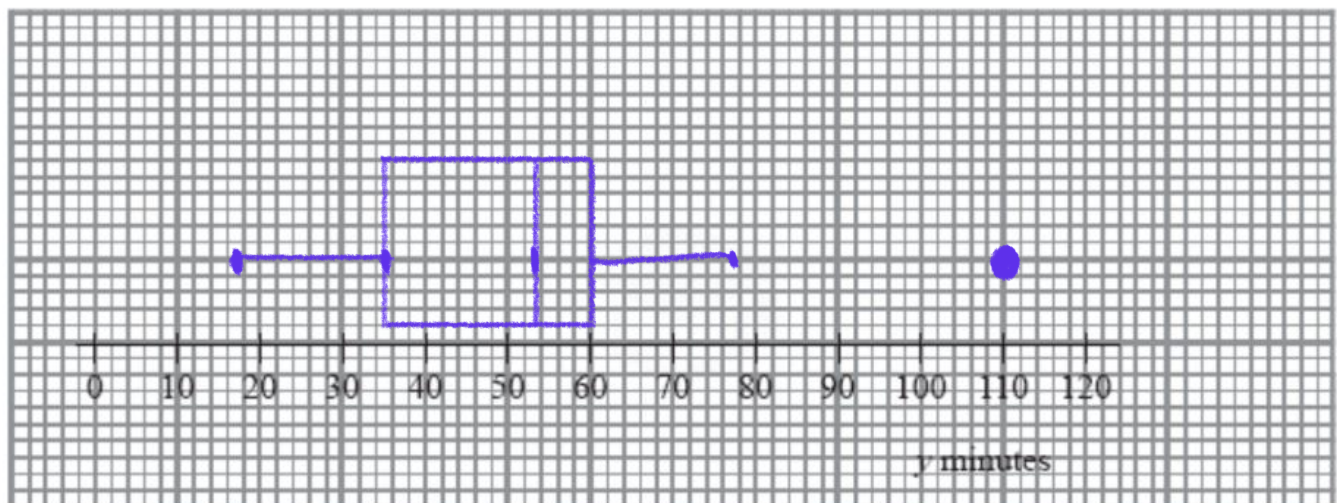
- (b) Show that 110 is the only outlier.

(2)

$$60 + 1.5 \times 25 = 97.5 < 110$$

So 110 is an outlier.

- (c) Using the graph below draw a box plot for these data indicating clearly the position of the outlier.

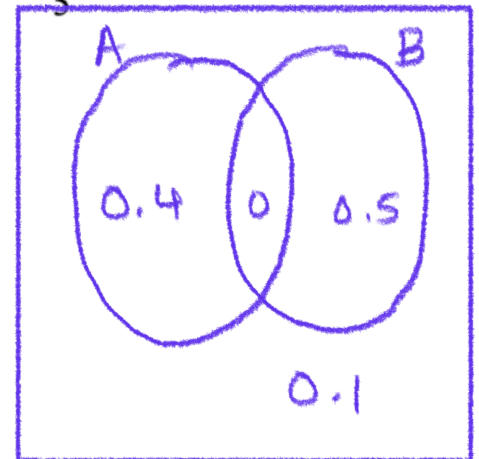


(3)

### Question 3

The events  $A$  and  $B$  are such that  $P(A) = \frac{2}{5}$ ,  $P(B) = \frac{1}{2}$  and  $P(A|B') = \frac{4}{5}$ .

Hosted on [revisely.co.uk](https://www.revisely.co.uk)



(a) Find

(i)  $P(A \cap B')$ , 0.4

(ii)  $P(A \cap B)$ , 0

(iii)  $P(A \cup B)$ , 0.9

(iv)  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0.5} = 0$

(7)

(b) State, with a reason, whether or not  $A$  and  $B$  are

(i) mutually exclusive,

Since  $P(A \cap B) = 0$ , then  $A$  and  $B$  are mutually exclusive.

(2)

(ii) independent.

$$P(A \cap B) = 0$$

$$P(A) \times P(B) = 0.4 \times 0.5 = 0.2$$

(2)

Since  $P(A \cap B) \neq P(A) \times P(B)$  then  $A$  and  $B$  are not independent.

#### Question 4

- (a) State two conditions under which a random variable can be modelled by a binomial distribution. revisely.co.uk

(1) Fixed no of trials, Independent trials, only two outcomes ie success or failure (2)

In the production of a certain electronic component it is found that 10% are defective.

The component is produced in batches of 20.

- (b) Write down a suitable model for the distribution of defective components in a batch. (1)

$$X \sim B(20, 0.1)$$

Find the probability that a batch contains

- (c) no defective components,

$$P(X=0) = 0.1216$$

Binomial PD

$$X = 0$$

$$N = 20$$

$$P = 0.1$$

- (d) more than 6 defective components. (2)

$$\begin{aligned} P(X > 6) &= 1 - P(X \leq 6) \\ &= 1 - 0.9976... = 0.002386 \end{aligned}$$

- (e) Find the mean and the variance of the defective components in a batch. (2)

$$\begin{aligned} E(X) &= np = 20 \times 0.1 = 2 \\ \text{Var}(X) &= npq = 20 \times 0.1 \times 0.9 = 1.8 \end{aligned}$$

A supplier buys 100 components. The supplier will receive a refund if there are more than 15 defective components.

- (f) Using a suitable approximation, find the probability that the supplier will receive a refund. (4)

$$X \sim B(100, 0.1)$$

$$X \sim N(10, 10)$$

$$\begin{aligned} P(X > 15.5) &= 1 - P(Z < 1.74) \\ &= \underline{\underline{0.0409}} \end{aligned}$$



### Question 5

Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly.

(Total 7 marks)

$$H_0: p = 0.3$$

$$H_1: p > 0.3$$

$$X \sim B(40, 0.3)$$

$$P(X \geq 18) = 1 - P(X \leq 17)$$

$= 0.032 < 0.05$ , reject  $H_0$ , hence the new fertiliser has increased the probability of a tomato being greater than 4cm.

$$P(X \geq 18) = 1 - P(X \leq 17) = 0.032$$

$$P(X \geq 17) = 1 - P(X \leq 16) = 0.0633$$

$$CR: X \geq 18$$

## Question 6

From past records a manufacturer of ceramic plant pots knows that 20% of them will have defects. To monitor the production process, a random sample of 25 pots is checked each day and the number of pots with defects is recorded.

- (a) Find the critical regions for a two-tailed test of the hypothesis that the probability that a plant pot has defects is 0.20. The probability of rejection in either tail should be as close as possible to 2.5%.

(5)

- (b) Write down the significance level of the above test.

(1)

A garden centre sells these plant pots at a rate of 10 per week. In an attempt to increase sales, the price was reduced over a six-week period. During this period a total of 74 pots was sold.

- (c) Using a 5% level of significance, test whether or not there is evidence that the rate of sales per week has increased during this six-week period.

(7)