AS STATISTICS PRACTICE 1

Question 1

A travel agent sells holidays from his shop. The price, in £, of 15 holidays sold on a particular day are shown below.

299	1050	2315	999	485
350	169	1015	650	830
99	2100	689	550	475

$\Sigma x = 12075$

$\Sigma x^2 = 15499685$

For these data, find

(a) the mean and the standard deviation,

(3)

$\bar{x} = \frac{12075}{15} = 805$

$sd = \sqrt{\frac{15499685}{15} - 805^2} = 621$

(b) the median and the inter-quartile range.

(4)

99, 169, 299, 350, 475, 485, 550, 650, 689, 830, 999, 1015, 1050, 2100, 2315

$Q_2 = 650$

$IQR = 1015 - 350 = 665$

An outlier is an observation that falls either more than 1.5 × (inter-quartile range) above the upper quartile or more than 1.5 × (inter-quartile range) below the lower quartile.

(c) Determine if any of the prices are outliers.

(3)

$Q_3 + 1.5(Q_3 - Q_1) = 1015 + 1.5 \times 665 = 2012.5$

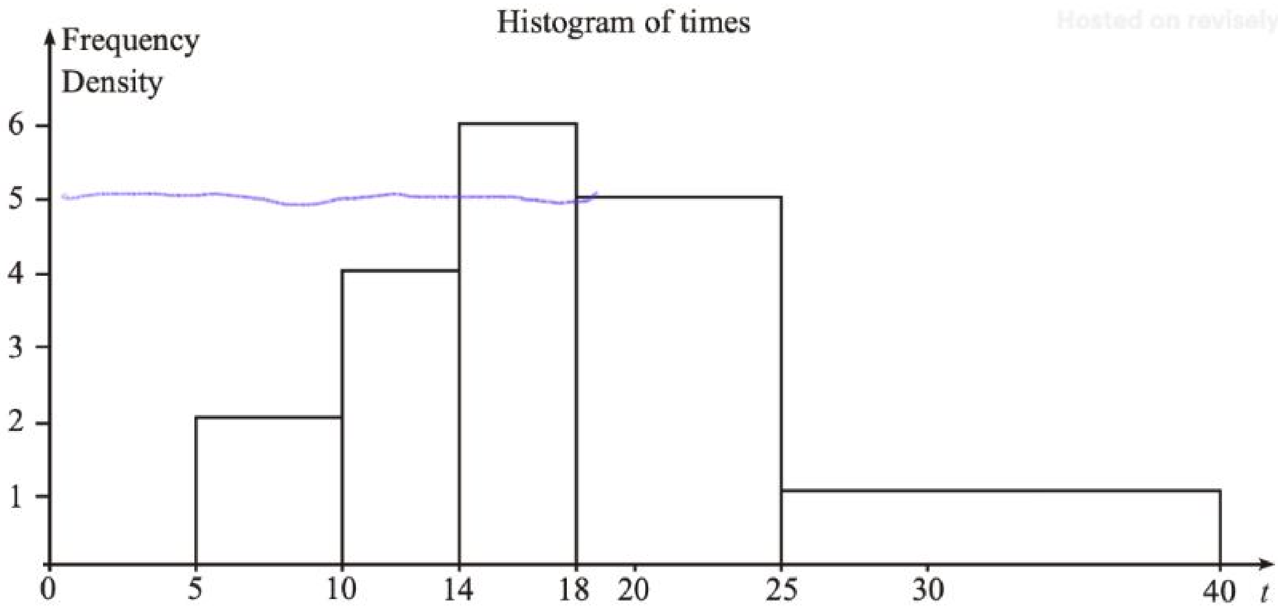
$Q_1 - 1.5(Q_3 - Q_1) = 350 - 1.5 \times 665 < 0$

$\therefore$ 2100 and 2315 are outliers

(e) Compare and contrast sales from the shop and sales from the website.

(2)

Question 2



The diagram above shows a histogram for the variable t which represents the time taken, in minutes, by a group of people to swim 500m.

(a) Complete the frequency table for t.

7.5 12 16 21.5 32.5

t	5–10	10–14	14–18	18–25	25–40
Frequency	10	16	24	35	15

(2)

(b) Estimate the number of people who took longer than 20 minutes to swim 500m. **(2)**

$(25-20)\times 5 + (40-25)\times 1 = 40$

(c) Find an estimate of the mean time taken. **(4)**

$$\frac{\Sigma fx}{\Sigma f} = \frac{1891}{100}$$

$$= \underline{\underline{18.91}}$$

(d) Find an estimate for the standard deviation of t.

(3)

$$\sigma = \sqrt{\frac{41033}{100} - 18.9^2}$$

$$= 7.26$$

(e) Find the median and quartiles for t.

Cu

(4)

$$Q_2 = 18$$

$$Q_1 = 10 + \frac{15}{16} \times 4 = 13.8125$$

$$Q_3 = 18 + \frac{25}{35} \times 7 = 23.15$$

Question 3

A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of one another.

(a) Find the probability that Linda scores 30 points in a round. **(2)**

$P(\text{Score } 30) = P(\text{hit, hit, hit}) = 0.6^3 = 0.216 \left(\frac{27}{125}\right)$

The random variable X is the number of points Linda scores in a round.

$\left\{\begin{matrix} 400 \\ 100 \end{matrix}\right\}$

(b) Find the probability distribution of X. **(5)**

x	0	10	20	30
	0.4	0.6×0.4	$0.6^2 \times 0.4$	
$P(X=x)$	0.4	0.24	0.144	0.216

(c) Find the mean and the standard deviation of X. **(5)**

$E(X) = (0 \times 0.4) + (10 \times 0.24) + (20 \times 0.144) + (30 \times 0.216)$
$= 11.76$

$E(X^2) = (10^2 \times 0.24) + (20^2 \times 0.144) + (30^2 \times 0.216)$
$= 276$

$Sd = \sqrt{E(X^2) - [E(X)]^2} = \sqrt{276 - 11.76^2} = 11.7$

A game consists of 2 rounds.

(d) Find the probability that Linda scores more points in round 2 than in round 1. **(6)**

P(Linda scores more in R2 than 1)

$= P(X_1 = 0 \text{ and } X_2 = 10, 20, 30) + P(X_1 = 10, X_2 = 20, 30)$

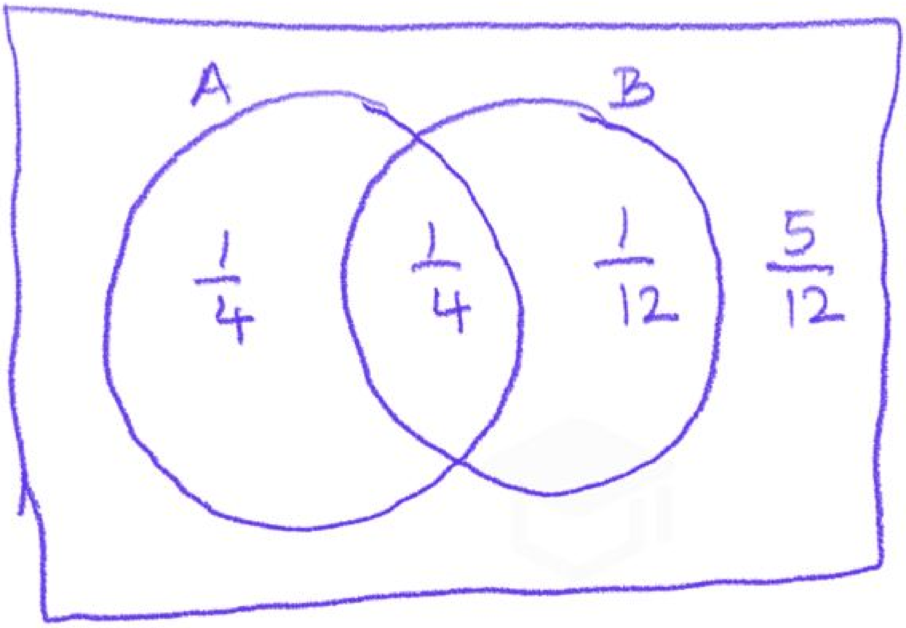
$P(X_1 = 20 + X_2 = 30) = 0.4 \times (0.24 + 0.144 + 0.216)$
$= 0.24$

$+ 0.24 \times (0.144 + 0.216) = 0.0864$
$+ (0.144 \times 0.216) = 0.031104$
$= 0.357504$

Question 4

The events A and B are such that $\mathrm{P}(A) = \frac{1}{2}$, $\mathrm{P}(B) = \frac{1}{3}$ and $\mathrm{P}(A \cap B) = \frac{1}{4}$.

(a) Using the space below, represent these probabilities in a Venn diagram.



(4)

Hence, or otherwise, find

(b) $\mathrm{P}(A \cup B)$, $= \frac{7}{12}$ (0.583) **(1)**

(c) $\mathrm{P}(A \mid B') = \dfrac{P(A \cup B)}{P(B')} = \dfrac{\frac{1}{4}}{\frac{2}{3}} = \dfrac{3}{8}$ **(2)**

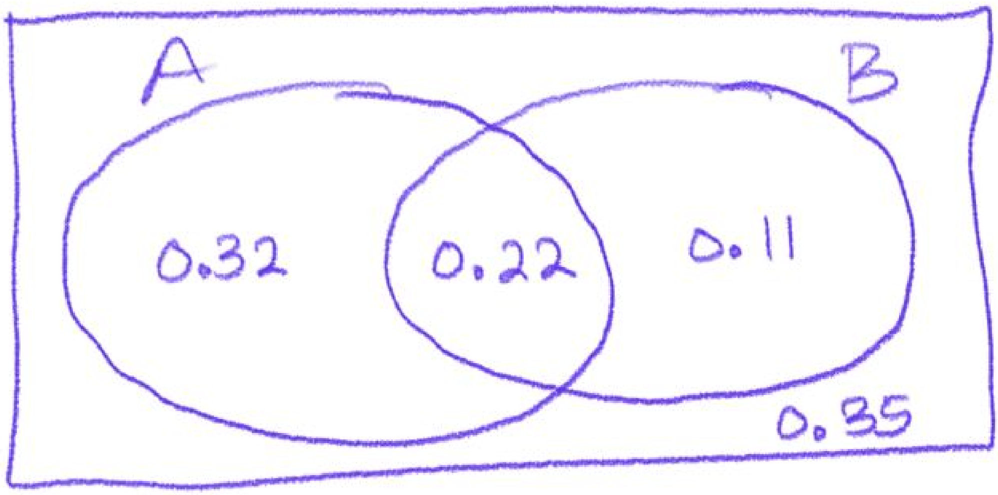
$= 0.375$

Question 5

For the events A and B,

$$P(A \cap B') = 0.32, \qquad P(A' \cap B) = 0.11 \qquad \text{and} \qquad P(A \cup B) = 0.65$$

(a) Draw a Venn diagram to illustrate the complete sample space for the events A and B. **(3)**



(b) Write down the value of P(A) and the value of P(B). **(3)**

$P(A) = 0.32 + 0.22 = 0.54$

$P(B) = 0.11 + 0.22 = 0.33$

(c) Find P($A \mid B'$). $= \dfrac{P(A \cap B)}{P(B')} = \dfrac{32}{67}$ **(2)**

(d) Determine whether or not A and B are independent. **(3)**

Independence: $P(A \cap B) = P(A) \times P(B)$

$0.22 \neq 0.54 \times 0.33$

Not independent

Question 6

A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.

A random sample of 10 patients with depression is taken from a doctor's records.

(a) Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug.

(2)

$X \sim B(10, 0.75)$

Given that the claim is correct,

(b) find the probability that the treatment will be successful for exactly 6 patients.

(2)

$P(X = 6) = 0.146$

Binomal PD
$x = 6$
$N = 10$
$p = 0.75$

The doctor believes that the claim is incorrect and the percentage who will recover is lower. From her records she took a random sample of 20 patients who had been treated with the new drug. She found that 13 had recovered.

(c) Stating your hypotheses clearly, test, at the 5% level of significance, the doctor's belief.

(6)

$H_0 : p = 0.75$
$H_1 : p < 0.75$

Under H_0: $X \sim B(20, 0.75)$

$P(X \leq 13) = 0.214$

$0.214 > 0.05$

Insufficient evidence to reject H_0, therefore doctor's belief is not supported by the sample.

(d) From a sample of size 20, find the greatest number of patients who need to recover from the test in part (c) to be significant at the 1% level.

(4)

$P(X \leq c) \leq 0.01$ for $p = 0.75$

$P(X \leq 9) = 0.0039 < 0.01$

$P(X \leq 10) = 0.0139 > 0.01$

Critical Region is $\{0, 9\}$, so greatest no. is 9.

Question 7

The probability of a bolt being faulty is 0.3. Find the probability that in a random sample of 20 bolts there are

(a) exactly 2 faulty bolts, $X \sim B(20, 0.3)$

(2)

$P(X = 2) = 0.0278$ (Binomial PD)

(b) more than 3 faulty bolts.

(2)

$P(X > 3) = 1 - P(X \leq 3) = 1 - 0.1071$

$= 0.893$

These bolts are sold in bags of 20. John buys 10 bags.

(c) Find the probability that exactly 6 of these bags contain more than 3 faulty bolts.

(3)

$\frac{10!}{4!\,6!} = 0.014$

Question 8

Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week, with the new driver, the taxi is late 3 times.

You may assume that the number of times a taxi is late in a week has a Binomial distribution.

Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly.

(Total 7 marks)

$H_0: p = 0.2$

$H_1: p > 0.2$

$X \sim B(5, 0.2)$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.94208$

$= 0.0579$

Since $0.0579 > 0.05$

Accept H_0, insufficient evidence at 5% s.l that there is an increase in the number of times the driver is late

Therefore Linda's claim is not justified.