AS MATHEMATICS

GRAPHS AND TRANSFORMATIONS

NAME_____

Question 1

(a) Sketch the curve with equation

$$y = \frac{k}{x} \qquad x \neq 0$$

where k is a positive constant.

(2)

(b) Hence or otherwise, solve

$$\frac{16}{x} \leqslant 2$$

(3)

Question 2

In this question you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = 8 - 10x + 6x^2 - x^3$

The curve C_2 has equation $y = x^2 - 12x + 14$

(a) Verify that when $x = 1$ the curves C_1 and C_2 intersect.

(2)

The curves also intersect when $x = k$.

Given that $k < 0$

(b) use algebra to find the exact value of k.

(5)

Question 3

(a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

Question 4

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

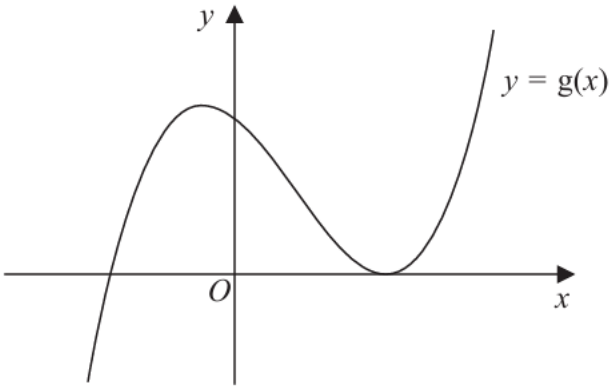


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

Question 5

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

Question 6

(a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

Question 7

The curve C_1 has equation

$$y = \frac{6}{x} + 3$$

(a) (i) Sketch C_1 stating the coordinates of any points where the curve cuts the coordinate axes.

(ii) State the equations of any asymptotes to the curve C_1

(3)

The curve C_2 has equation

$$y = 3x^2 - 4x - 10$$

(b) Show that C_1 and C_2 intersect when

$$3x^3 - 4x^2 - 13x - 6 = 0$$

(2)

Given that the x coordinate of one of the points of intersection is $-\frac{2}{3}$

(c) use algebra to find the x coordinates of the other points of intersection between C_1 and C_2

(*Solutions relying on calculator technology are not acceptable.*)

(4)

Question 8

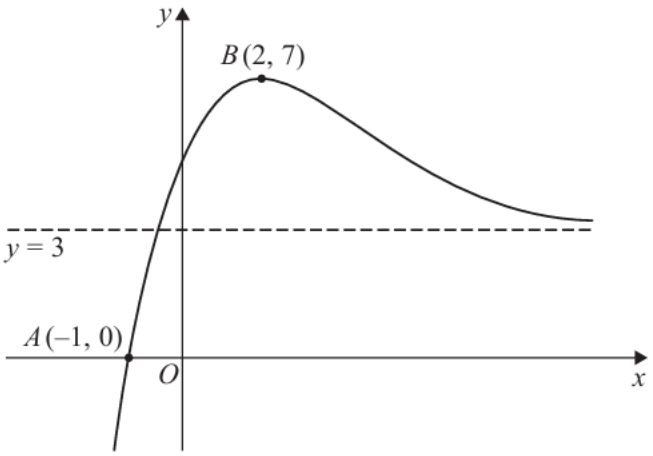


Figure 1

Figure 1 shows a curve with equation $y = f(x)$

The curve

- passes through the point $A(-1, 0)$
- has a maximum turning point at $B(2, 7)$
- has a horizontal asymptote with equation $y = 3$

On separate diagrams, sketch the curve with equation

(i) $y = f(x + 2)$

(3)

(ii) $y = -f(x)$

(3)

On each diagram, show clearly the coordinates of the points to which A and B are transformed and the equation of the asymptote.

Question 9

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{4 - x} \qquad x \neq 4$$

On your sketch show

- the equation of the vertical asymptote
- the coordinates of the point of intersection of C with the y-axis

(3)

The straight line l has equation $y = 2x + k$, where k is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$k^2 + 16k + 56 \geqslant 0$$

(4)

(ii) find the range of values of k, giving your answer in set notation.

(2)