

AS MATHEMATICS – ALGEBRA TEST

NAME Answer = 45 marks

Question 1

Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

$$\begin{aligned} \text{(i)} \quad \frac{16a^2}{a^{1/2}} &= 2 \\ 16a^{3/2} &= 2 \quad \checkmark \\ a^{3/2} &= \frac{1}{8} \quad \checkmark \\ a &= \left(\frac{1}{8}\right)^{2/3} \quad \checkmark \\ &= \frac{1}{4} \quad \checkmark \quad \textcircled{4} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \text{let } x &= b^2 \\ x^2 + 7x - 18 &= 0 \quad \checkmark \\ (x+9)(x-2) &= 0 \quad \checkmark \\ x &= -9, \quad x = 2 \quad \checkmark \\ b^2 &= -9, \quad b^2 = 2 \quad \textcircled{4} \\ b &= \pm\sqrt{2} \quad \checkmark \end{aligned}$$

Question 2

The line l has equation $y = -2x + 5$

- (a) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

- (b) Hence find the exact values of k for which l is a tangent to C .

(3)

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4(2)(k^2) = 0 \quad \checkmark$$

$$16 - 8k^2 = 0$$

$$8k^2 - 16 = 0 \quad \checkmark \quad \hookrightarrow$$

$$k^2 - 2 = 0$$

$$k^2 = 2 \quad \therefore \underline{k = \pm\sqrt{2}} \quad \checkmark \quad (3)$$

Question 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (i) Solve the equation

$$\sqrt{2} \times 9$$

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

- (ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

$$(i) \quad x\sqrt{2} - 3\sqrt{2} = x$$

$$x\sqrt{2} - x = 3\sqrt{2} \quad \checkmark$$

$$x(\sqrt{2} - 1) = 3\sqrt{2}$$

$$x = \frac{3\sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} \quad \checkmark$$

$$= \underline{6 + 3\sqrt{2}} \quad \checkmark \quad (3)$$

$$(ii) \quad 2\sqrt{2} \times 2^{6x-4} = 2^0 \quad \checkmark$$

$$2^1 \times 2^{\frac{1}{2}} \times 2^{6x-4} = 2^0$$

$$1 + \frac{1}{2} + 6x - 4 = 0 \quad \checkmark$$

$$6x - 2.5 = 0$$

$$6x = 2.5$$

$$x = \frac{5}{12} \quad \checkmark \quad (3)$$

Question 4

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

$$x^2 - x - 20 > 0$$

$$(x - 5)(x + 4) > 0$$

$$\therefore x = 5, x = -4$$



$$\therefore x < -4, x > 5 \Rightarrow \{x \in \mathbb{R} : x < -4\} \cup \{x \in \mathbb{R} : x > 5\}$$

Question 5

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x , writing your answer in simplest form.

(3)

$$\frac{3^{2x-2}}{3^{y+2}} = 3^4$$

$$3^{2x-2-(y+2)} = 3^4$$

$$2x - 2 - y - 2 = 4$$

$$2x - y - 4 = 4$$

$$2x - y = 8$$

$$\underline{y = 2x - 8}$$

(3)

Question 6

Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

$$x(3x^2 - 17x - 6) = 0$$

$$x(3x - 18)(3x + 1) = 0$$

$$x(x - 6)(3x + 1) = 0$$

$$x = 0, \quad x = 6, \quad x = -\frac{1}{3}$$

Question 7

Using the substitution $u = \sqrt{x}$ or otherwise, solve

$$6x + 7\sqrt{x} - 20 = 0$$

(4)

$$6u^2 + 7u - 20 = 0$$

$$\frac{(6u + 15)}{3} \cdot \frac{(6u - 8)}{2} = 0$$

$$(2u + 5)(3u - 4) = 0$$

$$u = -\frac{5}{2}, \quad u = \frac{4}{3}$$

$$\sqrt{x} = -\frac{5}{2}, \quad \sqrt{x} = \frac{4}{3}$$

$\sqrt{x} \geq 0$, so
underline.

$$x = \frac{16}{9}$$

(4)

Question 8

An equation

$$x^2 - 3px + 5q + 4 = 0$$

where p and q are constants, has equal roots.

Show that

$$q = \frac{1}{20}(3p+4)(3p-4)$$

(3)

$$b^2 - 4ac = 0$$
$$(-3p)^2 - 4(5q+4) = 0$$
$$9p^2 - 20q - 16 = 0$$
$$20q = 9p^2 - 16 \quad \checkmark$$

$$20q = (3p-4)(3p+4) \quad \checkmark$$

$$q = \frac{1}{20}(3p-4)(3p+4) \quad \checkmark \quad (3)$$

Question 9

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Find the values of x for which

(a) $4(x-3) > 8-2x$

(2)

(b) $(2x+5)(x-8) < 0$

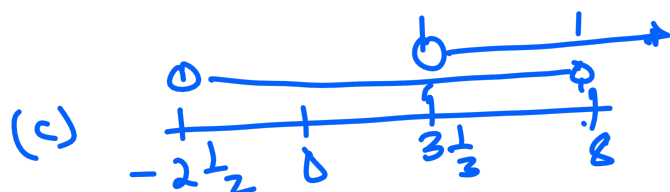
(2)

(c) both $4(x-3) > 8-2x$ and $(2x+5)(x-8) < 0$

(1)

$$(a) \quad 4x - 12 > 8 - 2x$$
$$6x > 20 \quad \checkmark \quad (2)$$
$$x > 3\frac{1}{3} \quad \checkmark$$

$$(b) \quad -2\frac{1}{2} < x < 8 \quad \checkmark \quad (2)$$



$$\therefore 3\frac{1}{3} < x < 8 \quad \checkmark \quad (1)$$

Question 10

$$y = \frac{1}{4-x}$$

The straight line l has equation $y = 2x + k$, where k is a constant.

Given that l cuts C at least once,

(i) show that

$$k^2 + 16k + 56 \geq 0 \quad (4)$$

(ii) find the range of values of k , giving your answer in set notation. (2)

$$(i) \quad \frac{1}{4-x} = 2x + k$$

$$1 = (2x + k)(4 - x) \quad \checkmark$$

$$1 = 8x - 2x^2 + 4k - kx$$

$$2x^2 + kx - 8x + 1 - 4k = 0 \quad \checkmark$$

$$2x^2 + (k - 8)x + 1 - 4k = 0$$

$$b^2 - 4ac \geq 0$$

$$(k - 8)^2 - 4(2)(1 - 4k) \geq 0 \quad \checkmark$$

$$k^2 - 16k + 64 - 8 + 32k \geq 0 \quad \checkmark$$

$$k^2 + 16k + 56 \geq 0 \quad \checkmark$$

(4)

$$(ii) \quad k = \frac{-16 \pm \sqrt{16^2 - 224}}{2}$$

$$= \frac{-16 \pm 4\sqrt{2}}{2} \quad \checkmark$$

$$= -8 \pm 2\sqrt{2} \quad \checkmark \quad (2) \quad \{x: x < -8 - 2\sqrt{2}\} \cup \{x: x > -8 + 2\sqrt{2}\}$$

Question 11

$f(x) = x^3 + x^2 + px + q$, where p and q are constants.

Given that $f(5) = 0$ and $f(-3) = 8$,

find the values of p and q ,

$$5^3 + 5^2 + 5p + q = 0 \Rightarrow 5p + q = -150 \quad (4)$$

$$(-3)^3 + (-3)^2 - 3p + q = 8 \Rightarrow 3p - q = -26$$

$$8p = -176$$

$$p = 22 \quad \checkmark$$

$$110 + q = -150$$

$$q = -260 \quad \checkmark$$

(4)