AS MATHEMATICS – MECHANICS

CONSTANT ACCELERATION

NAME_____

Formulae

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

Unless otherwise indicated, wherever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Question 1

A man throws a tennis ball into the air so that, at the instant when the ball leaves his hand, the ball is 2 m above the ground and is moving vertically upwards with speed $9\,\text{m s}^{-1}$

The motion of the ball is modelled as that of a particle moving freely under gravity and the acceleration due to gravity is modelled as being of constant magnitude $10\,\text{m s}^{-2}$

The ball hits the ground T seconds after leaving the man's hand.

Using the model, find the value of T.

(4)

Question 2

A train travels along a straight horizontal track between two stations, A and B.

In a model of the motion, the train starts from rest at A and moves with constant acceleration $0.3\,\text{m s}^{-2}$ for 80 s.
The train then moves at constant velocity before it moves with a constant deceleration of $0.5\,\text{m s}^{-2}$, coming to rest at B.

(a) For this model of the motion of the train between A and B,

(i) state the value of the constant velocity of the train,

(ii) state the time for which the train is decelerating,

(iii) sketch a velocity-time graph.

(3)

The total distance between the two stations is 4800 m.

(b) Using the model, find the total time taken by the train to travel from A to B.

(3)

(c) Suggest one improvement that could be made to the model of the motion of the train from A to B in order to make the model more realistic.

(1)

Question 3

At time $t = 0$, a parachutist falls vertically from rest from a helicopter which is hovering at a height of 550 m above horizontal ground.

The parachutist, who is modelled as a particle, falls for 3 seconds before her parachute opens.

While she is falling, and before her parachute opens, she is modelled as falling freely under gravity.

The acceleration due to gravity is modelled as being $10\,\text{m}\,\text{s}^{-2}$.

(a) Using this model, find the speed of the parachutist at the instant her parachute opens.

(1)

When her parachute is open, the parachutist continues to fall vertically.

Immediately after her parachute opens, she decelerates at $12\,\text{m}\,\text{s}^{-2}$ for 2 seconds before reaching a constant speed and she reaches the ground with this speed.

The total time taken by the parachutist to fall the 550 m from the helicopter to the ground is T seconds.

(b) Sketch a speed-time graph for the motion of the parachutist for $0 \leqslant t \leqslant T$.

(2)

(c) Find, to the nearest whole number, the value of T.

(5)

In a refinement of the model of the motion of the parachutist, the effect of air resistance is included before her parachute opens and this refined model is now used to find a new value of T.

(d) How would this new value of T compare with the value found, using the initial model, in part (c)?

(1)

(e) Suggest one further refinement to the model, apart from air resistance, to make the model more realistic.

(1)

Question 4

At time $t = 0$, a small ball is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from a point A that is 16.8 m above horizontal ground.

The speed of the ball at the instant immediately before it hits the ground for the first time is $19\,\text{m s}^{-1}$

The ball hits the ground for the first time at time $t = T$ seconds.

The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

Using the model,

(a) show that $U = 5$

(2)

(b) find the value of T,

(2)

(c) find the time from the instant the ball is projected until the instant when the ball is 1.2 m below A.

(4)

(d) Sketch a velocity-time graph for the motion of the ball for $0 \leqslant t \leqslant T$, stating the coordinates of the start point and the end point of your graph.

(2)

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

(e) State, with a reason, how this new value of U would compare with the value found in part (a), using the initial unrefined model.

(1)

(f) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.

(1)

Question 5

At time $t = 0$, a small stone is thrown vertically upwards with speed $14.7\,\text{m s}^{-1}$ from a point A.

At time $t = T$ seconds, the stone passes through A, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(a) find the value of T,

(2)

(b) find the total distance travelled by the stone in the first 4 seconds of its motion.

(4)

(c) State one refinement that could be made to the model, apart from air resistance, that would make the model more realistic.

(1)

Question 6

The point A is 1.8 m vertically above horizontal ground.

At time $t = 0$, a small stone is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from the point A.

At time $t = T$ seconds, the stone hits the ground.

The speed of the stone as it hits the ground is $10\,\text{m s}^{-1}$

In an initial model of the motion of the stone as it moves from A to where it hits the ground

- the stone is modelled as a particle moving freely under gravity
- **the acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$**

Using the model,

(a) find the value of U,

(3)

(b) find the value of T.

(2)

(c) Suggest one refinement, apart from including air resistance, that would make the model more realistic.

(1)

In reality the stone will not move freely under gravity and will be subject to air resistance.

(d) Explain how this would affect your answer to part (a).

(1)

Question 7

A train travels along a straight horizontal track from station P to station Q.

In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of $25\,\text{m s}^{-1}$

The train then travels at this constant speed of $25\,\text{m s}^{-1}$ before finally moving with constant deceleration until it comes to rest at Q.

The time spent decelerating is four times the time spent accelerating.

The journey from P to Q takes 700 s.

Using the model,

(a) sketch a speed-time graph for the motion of the train between the two stations P and Q.

(1)

The distance between the two stations is 15 km.

Using the model,

(b) show that the time spent accelerating by the train is 40 s,

(3)

(c) find the acceleration, in m s^{-2}, of the train,

(1)

(d) find the speed of the train 572 s after leaving P.

(2)

(e) State one limitation of the model which could affect your answers to parts (b) and (c).

(1)

Question 8

A small stone is projected vertically upwards with speed $39.2\,\text{m s}^{-1}$ from a point O.

The stone is modelled as a particle moving freely under gravity from when it is projected until it hits the ground 10 s later.

Using the model, find

(a) the height of O above the ground,

(3)

(b) the total length of time for which the speed of the stone is less than or equal to $24.5\,\text{m s}^{-1}$

(3)

(c) State one refinement that could be made to the model that would make your answer to part (a) more accurate.

(1)