KINEMATICS – VARIABLE ACCELERATION

AS MATHEMATICS – EDEXCEL

NAME _____

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Mathematical formulae

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

Unless otherwise indicated, wherever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Question 1

A particle, P, moves along the x-axis. At time t seconds, $t \geqslant 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

$x = \frac{1}{2}t^4 - t^3 + \frac{1}{2}t^2$

$\frac{dx}{dt} = v = 2t^3 - 3t^2 + t = 0$

$t(2t^2 - 3t + 1) = 0$

$t(t-1)(2t-1) = 0$

$t = 0, \quad t = 1, \quad t = \frac{1}{2}$

(b) Find the total distance travelled by P in the time interval $0 \leqslant t \leqslant 2$

(3)

$t = 0: \quad x = \frac{1}{2}(0)^2[0^2 - 2(0) + 1] = 0$

$t = 0.5: \quad x = \frac{1}{2}(\frac{1}{2})^2[(\frac{1}{2})^2 - 2(\frac{1}{2}) + 1] = \frac{1}{32}\text{m}$

$t = 1: \quad x = \frac{1}{2}(1)^2[(1)^2 - 2(1) + 1] = 0\text{m}$ → back to origin

$t = 2: \quad x = \frac{1}{2}(2)^2[(2)^2 - 2(2) + 1] = 2\text{m}$

Total distance = 2.0625m.

(c) Show that P will never move along the negative x-axis.

(2)

$x = \frac{1}{2}t^2(t^2 - 2t + 1) = \frac{1}{2}t^2(t-1)^2$

as t^2 and $(t-1)^2$ are perfect squares, ie $x \geqslant 0$

∴ P will never move in negative x-axis.

Question 2

A particle, P, moves along a straight line such that at time t seconds, $t \geqslant 0$, the velocity of P, $v\,\text{m s}^{-1}$, is modelled as

$$v = 12 + 4t - t^2$$

Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

$0 = 12 + 4t - t^2$

$t^2 - 4t - 12 = 0$

$(t - 6)(t + 2) = 0$

$t = 6, \quad t = -2$

$\therefore \quad t = 6$

$a = \frac{dv}{dt} = 4 - 2t$

$= 4 - 2 \times 6$

$= -8\,\text{m s}^{-2}$

$|a| = 8\,\text{m s}^{-2}$

(b) the distance travelled by P in the interval $0 \leqslant t \leqslant 3$

(3)

$s = \int v\,dt$

$s = \int_0^3 12 + 4t - t^2$

$= \left[12t + 2t^2 - \frac{1}{3}t^3\right]_0^3$

$= 12(3) + 2(3)^2 - \frac{1}{3}(3)^3 - 0$

$= 45\text{m}$

Question 3

A particle P moves along a straight line such that at time t seconds, $t \geqslant 0$, after leaving the point O on the line, the velocity, $v\,\text{m s}^{-1}$, of P is modelled as

$$v = (7 - 2t)(t + 2)$$

(a) Find the value of t at the instant when P stops accelerating. **(4)**

$a = 0$

$v = (7-2t)(t+2) = -2t^2 + 3t + 14$

$a = \frac{dv}{dt} = -4t + 3 = 0$

$4t = 3$

$t = \frac{3}{4}$

(b) Find the distance of P from O at the instant when P changes its direction of motion. **(5)**

In this question, solutions relying on calculator technology are not acceptable.

(b) $v = 0 : (7-2t)(t+2) = 0$

$t = -2, \quad t = 3.5$

$\therefore t = 3.5$

Changes direction of motion when $t = 3.5$ sec.

$r = \int v\,dt$

$= \int_0^{3.5} -2t^2 + 3t + 14 \; dt$

$= \left[\frac{-2t^3}{3} + \frac{3t^2}{2} + 14t\right]_0^{3.5}$

$= 38.8\text{ m}$

Question 4

A particle P moves along a straight line.

At time t seconds, the velocity $v\,\text{m s}^{-1}$ of P is modelled as

$$v = 10t - t^2 - k \qquad t \geqslant 0$$

where k is a constant.

(a) Find the acceleration of P at time t seconds.

(2)

$a = \frac{dv}{dt} = 10 - 2t$

The particle P is instantaneously at rest when $t = 6$

(b) Find the other value of t when P is instantaneously at rest.

(4)

$t=6,\ v=0: \quad 0 = 10(6) - (6)^2 - k$

$k = 24$

$\therefore v = 10t - t^2 - 24$

$v=0: \quad 0 = 10t - t^2 - 24$

$t^2 - 10t + 24 = 0$

$(t-4)(t-6) = 0$

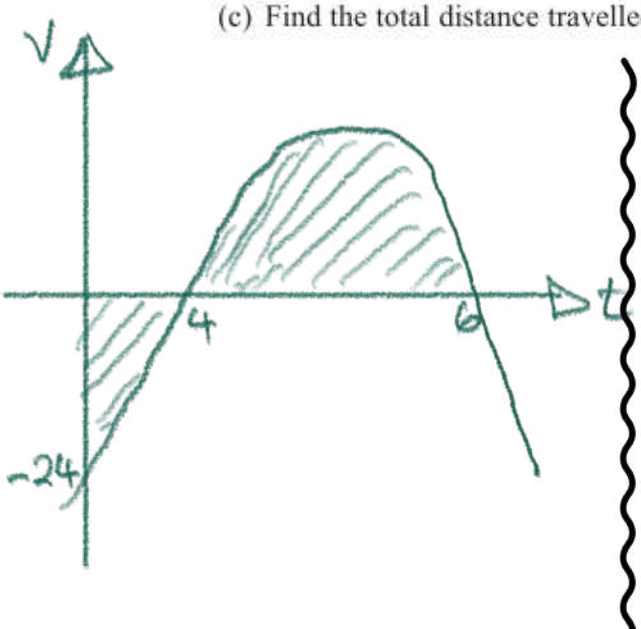
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$t = 4,\ t = 6$

$\therefore$ t = 4 seconds

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 6$

(4)



$\int_0^4 10t - t^2 - 24 \Big] + \Big[\int_4^6 10t - t^2 - 24 \Big]$

$5t^2 - \frac{1}{3}t^3 - 24t \Big]_0^4 + \Big[5t^2 - \frac{1}{3}t^3 - 24t \Big]_4^6$

$= \frac{116}{3}$

$= 38.7\text{m}$

Question 5

A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest.

(6)

$v = \frac{ds}{dt} = t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$

$t = 3, t = 2$

$a = \frac{dv}{dt} = 2t - 5$

When $t = 2$: $a = 2(2) - 5$

$= -1 \text{ ms}^{-2}$

$t = 3$: $a = 2(3) - 5$

$= 1 \text{ ms}^{-2}$

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

$t = 2$: $s = \frac{1}{3}(2)^3 - \frac{5}{2}(2)^2 + 6(2) = \frac{14}{3}$

$t = 3$: $s = \frac{1}{3}(3)^3 - \frac{5}{2}(3)^2 + 6(3) = \frac{9}{2}$

$t = 4$: $s = \frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 6(4) = \frac{16}{3}$

Total distance $= \frac{14}{3} + \left(\frac{14}{3} - \frac{9}{2}\right) + \left(\frac{16}{3} - \frac{9}{2}\right)$

$= \frac{17}{3} = 5.67\text{m}.$

Question 6

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A fixed point O lies on a straight line.

A particle P moves along the straight line such that at time t seconds, $t \geqslant 0$, after passing through O, the velocity of P, $v\,\text{m s}^{-1}$, is modelled as

$$v = 15 - t^2 - 2t$$

(a) Verify that P comes to instantaneous rest when $t = 3$ **(1)**

$v = 0$

$t^2 + 2t - 15 = 0$

$(t + 5)(t - 3) = 0$ $\rightarrow$ $t = -5,\ t = 3$

$\therefore\ t = 3$

(b) Find the magnitude of the acceleration of P when $t = 3$ **(3)**

$$a = \frac{dv}{dt} = -2t - 2$$

at $t = 3$: $-2(3) - 2$

$= -8\,\text{m s}^{-2}$

$a = 8\,\text{m s}^{-2}$

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$ **(4)**

$$s = \int v\,dt =$$

$$s = \int_0^3 (15 - t^2 - 2t)\,dt + \int_3^4 (15 - t^2 - 2t)\,dt$$

$$= \left[15t - \frac{1}{3}t^3 - t^2\right]_0^3 + \left[15t - \frac{1}{3}t^3 - t^2\right]_3^4$$

$$= 27 + \left|-\frac{13}{3}\right|$$

$$= \frac{94}{3}\,\text{m}$$

Question 7

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A particle is moving along a straight line.

At time t seconds, $t > 0$, the velocity of the particle is $v\,\text{ms}^{-1}$, where

$$v = 2t - 7\sqrt{t} + 6 \qquad = 2t - 7t^{\frac{1}{2}} + 6$$

(a) Find the acceleration of the particle when $t = 4$

(3)

When $t = 1$ the particle is at the point X.

When $t = 2$ the particle is at the point Y.

$$a = \frac{dv}{dt} = 2 - \frac{7}{2}t^{-\frac{1}{2}}$$

when $t=4$: $a = 2 - \dfrac{7}{2\sqrt{4}}$

$$= 0.25\,\text{ms}^{-2}$$

Given that the particle does not come to instantaneous rest in the interval $1 < t < 2$

(b) show that $XY = \frac{1}{3}\left(41 - 28\sqrt{2}\right)$ metres.

(4)

$$XY = \int_1^2 2t - 7t^{\frac{1}{2}} + 6\,dt$$

$$= t^2 - \frac{7\times 2}{3}t^{\frac{3}{2}} + 6t\Big]_1^2$$

$$= t^2 - \frac{14}{3}t^{\frac{3}{2}} + 6t\Big]_1^2$$

$$= 2^2 - \frac{14}{3}(2)^{\frac{3}{2}} + 6(2)\Big] - \Big[1 - \frac{14}{3} + 6\Big]$$

$$= \frac{1}{3}\left(41 - 28\sqrt{2}\right)$$