A LEVEL MATHEMATICS

STATISTICS YEAR 2 – NORMAL DISTRIBUTION

NAME _____

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Question 1

Fen makes cheese which is sold in packs.
The weight, in grams, of each pack is normally distributed with mean 540 and standard deviation σ

Fen knows that 80% of the packs have weights between 530 g and 550 g

(a) Find an estimate for the value of σ
Give your answer to 3 decimal places.

(3)

Fen makes 250 packs of this cheese.

(b) Estimate the expected number of these packs that weigh less than 530 g

(2)

Fen also makes yoghurt which is sold in pots.

The volume of yoghurt in a pot is normally distributed with mean 275 ml and standard deviation 11 ml

(c) Find the probability that a randomly chosen pot contains more than 263 ml of yoghurt.

(1)

Fen buys a new machine to put the yoghurt into the pots.

The standard deviation of the volume of yoghurt in each pot is unchanged at 11 ml
However, Fen believes that the mean volume may have changed from 275 ml

A random sample of 36 pots is taken and the mean volume of yoghurt in these pots is 271.7 ml

(d) Carry out a suitable test to assess Fen's belief.
You should

- state your hypotheses clearly
- use a 5% level of significance
- state your conclusion clearly

(4)

Question 2

The heights of men in a tennis club are normally distributed with a mean of 183 cm and a standard deviation of 4.9 cm.

A man from the tennis club is selected at random.

(a) Find the probability that the man's height is

(i) more than 186 cm,

(ii) between 175 cm and 185 cm.

(2)

The heights of women in the tennis club are normally distributed with a mean of μ cm and a standard deviation of σ cm.

Given that 40% of the women are shorter than 170 cm and 15% are taller than 175 cm,

(b) find the value of μ and the value of σ
Show your working clearly.

(5)

A man from the tennis club and a woman from the tennis club are selected at random.

(c) Find the probability that both players have heights between 170 cm and 175 cm.

(3)

Question 3

George throws a ball at a target 15 times.
Each time George throws the ball, the probability of the ball hitting the target is 0.48

The random variable X represents the number of times George hits the target in 15 throws.

(a) Find

(i) $P(X = 3)$

(ii) $P(X \geqslant 5)$

(3)

George now throws the ball at the target 250 times.

(b) Use a normal approximation to calculate the probability that he will hit the target more than 110 times.

(3)

Question 4

The records for a school athletics club show that the height, H metres, achieved by students in the high jump is normally distributed with mean 1.4 metres and standard deviation 0.15 metres.

(a) Find the proportion of these students achieving a height of more than 1.6 metres.

(1)

The records also show that the time, T seconds, to run 1500 metres is normally distributed with mean 330 seconds and standard deviation 26 seconds.

The school's Head would like to use these distributions to estimate the proportion of students from the school athletics club who can jump higher than 1.6 metres **and** can run 1500 metres in less than 5 minutes.

(b) State a necessary assumption about H and T for the Head to calculate an estimate of this proportion.

(1)

(c) Find the Head's estimate of this proportion.

(3)

Students in the school athletics club also throw the discus.

The random variable $D \sim \mathrm{N}(\mu, \sigma^2)$ represents the distance, in metres, that a student can throw the discus.

Given that $\mathrm{P}(D < 16.3) = 0.30$ and $\mathrm{P}(D > 29.0) = 0.10$

(d) calculate the value of μ and the value of σ

(5)

Question 5

A manufacturer uses a machine to make metal rods.

The length of a metal rod, L cm, is normally distributed with

- a mean of 8 cm
- a standard deviation of x cm

Given that the proportion of metal rods less than 7.902 cm in length is 2.5%

(a) show that $x = 0.05$ to 2 decimal places.

(2)

(b) Calculate the proportion of metal rods that are between 7.94 cm and 8.09 cm in length.

(1)

The **cost** of producing a single metal rod is 20p

A metal rod

- where $L < 7.94$ is **sold** for scrap for 5p
- where $7.94 \leqslant L \leqslant 8.09$ is **sold** for 50p
- where $L > 8.09$ is shortened for an extra **cost** of 10p and then **sold** for 50p

(c) Calculate the expected profit per 500 of the metal rods.
Give your answer to the nearest pound.

(5)

The same manufacturer makes metal hinges in large batches.

The hinges each have a probability of 0.015 of having a fault.

A random sample of 200 hinges is taken from each batch and the batch is accepted if fewer than 6 hinges are faulty.

The manufacturer's aim is for 95% of batches to be accepted.

(d) Explain whether the manufacturer is likely to achieve its aim.

(4)

Question 6

The heights of females from a country are normally distributed with

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

Given that 1% of females from this country are shorter than k cm,

(a) find the value of k

(2)

(b) Find the proportion of females from this country with heights between 150 cm and 175 cm

(1)

A female, from this country, is chosen at random from those with heights between 150 cm and 175 cm

(c) Find the probability that her height is more than 160 cm

(4)

The heights of females from a different country are normally distributed with a standard deviation of 7.4 cm

Mia believes that the mean height of females from this country is less than 166.5 cm

Mia takes a random sample of 50 females from this country and finds the mean of her sample is 164.6 cm

(d) Carry out a suitable test to assess Mia's belief.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

Question 7

A health centre claims that the time a doctor spends with a patient can be modelled by a normal distribution with a mean of 10 minutes and a standard deviation of 4 minutes.

(a) Using this model, find the probability that the time spent with a randomly selected patient is more than 15 minutes.

(1)

Some patients complain that the mean time the doctor spends with a patient is more than 10 minutes.

The receptionist takes a random sample of 20 patients and finds that the mean time the doctor spends with a patient is 11.5 minutes.

(b) Stating your hypotheses clearly and using a 5% significance level, test whether or not there is evidence to support the patients' complaint.

(4)

The health centre also claims that the time a dentist spends with a patient during a routine appointment, T minutes, can be modelled by the normal distribution where $T \sim \mathrm{N}(5, 3.5^2)$

(c) Using this model,

(i) find the probability that a routine appointment with the dentist takes less than 2 minutes

(1)

(ii) find $\mathrm{P}(T < 2 \mid T > 0)$

(3)

(iii) hence explain why this normal distribution may not be a good model for T.

(1)

The dentist believes that she cannot complete a routine appointment in less than 2 minutes.

She suggests that the health centre should use a refined model only including values of $T > 2$

(d) Find the median time for a routine appointment using this new model, giving your answer correct to one decimal place.

(5)

Question 8

A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml

Given that 15% of bottles contain less than 24.63 ml

(a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$

(5)

A random sample of 200 bottles is taken.

(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml

(3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml

(c) Test Hannah's belief at the 5% level of significance.
You should state your hypotheses clearly.

(5)