A LEVEL MATHEMATICS

STATISTICS YEAR 1/2 – STATISTICAL DISTRIBUTIONS

NAME _____

Discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$

Sampling distributions

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$$

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Question 1

Sala has a biased coin for which the probability of a head is $\frac{2}{5}$

In a game, Sala spins the coin repeatedly and stops when

either the 2nd head is obtained
or the coin has been spun 5 times

The random variable X represents the number of times Sala spins the coin in one game.

(a) Explain why a binomial distribution is not an appropriate model for X

(1)

(b) Show that $P(X = 4) = \frac{108}{625}$

(3)

(c) Find the probability distribution of X

(5)

The random variable H represents the number of heads obtained in one game.

(d) Find $P(H = 1)$

(2)

(e) Write down $P(\{H = 1\} \mid \{X = 4\})$

(1)

Using your answer to part (e)

(f) state the relationship between the event $\{H = 1\}$ and the event $\{X = 4\}$

(1)

Question 2

A manufacturer makes components for the car industry.

On average 1.5% of the components made are defective.

(a) Using a suitable model, find the probability that in a random sample of 36 components there are

(i) exactly 2 defective components,

(ii) more than 3 defective components.

(3)

Once every six months, the manufacturer tests whether the proportion of defective components has changed using hypotheses $H_0 : p = 0.015$ and $H_1 : p \neq 0.015$

A random sample of 260 components is taken and 8 are found to be defective.

(b) Using a 5% level of significance, complete the test.

(3)

Question 3

The discrete random variable R takes even integer values from 2 to $2n$ inclusive.

The probability distribution of R is given by

$$\mathrm{P}(R = r) = \frac{r}{k} \qquad r = 2, 4, 6, \ldots, 2n$$

where k is a constant.

(a) Show that $k = n(n + 1)$

(4)

When $n = 20$

(b) find the exact value of $\mathrm{P}(16 \leqslant R < 26)$

(2)

When $n = 20$, a random value g of R is taken and the quadratic equation in x

$$x^2 + gx + 3g = 5$$

is formed.

(c) Find the exact probability that the equation has no real roots.

(5)

Question 4

A machine fills packets with sweets and $\frac{1}{7}$ of the packets also contain a prize.

The packets of sweets are placed in boxes before being delivered to shops.
There are 40 packets of sweets in each box.

The random variable T represents the number of packets of sweets that contain a prize in each box.

(a) State a condition needed for T to be modelled by $\mathrm{B}(40, \frac{1}{7})$

(1)

A box is selected at random.

(b) Using $T \sim \mathrm{B}(40, \frac{1}{7})$ find

(i) the probability that the box has exactly 6 packets containing a prize,

(ii) the probability that the box has fewer than 3 packets containing a prize.

(2)

Kamil's sweet shop buys 5 boxes of these sweets.

(c) Find the probability that exactly 2 of these 5 boxes have fewer than 3 packets containing a prize.

(2)

Kamil claims that the proportion of packets containing a prize is less than $\frac{1}{7}$

A random sample of 110 packets is taken and 9 packets contain a prize.

(d) Use a suitable test to assess Kamil's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

(4)

Question 5

Tisam is playing a game.
She uses a ball, a cup and a spinner.

The random variable X represents the number the spinner lands on when it is spun.
The probability distribution of X is given in the following table

x	20	50	80	100
$P(X = x)$	a	b	c	d

where a, b, c and d are probabilities.

To play the game

- the spinner is spun to obtain a value of x
- Tisam then stands x cm from the cup and tries to throw the ball into the cup

The event S represents the event that Tisam successfully throws the ball into the cup.

To model this game Tisam assumes that

- $P(S \mid \{X = x\}) = \frac{k}{x}$ where k is a constant
- $P(S \cap \{X = x\})$ should be the same whatever value of x is obtained from the spinner

Using Tisam's model,

(a) show that $c = \frac{8}{5}b$

(2)

(b) find the probability distribution of X

(5)

Nav tries, a large number of times, to throw the ball into the cup from a distance of 100 cm.
He successfully gets the ball in the cup 30% of the time.

(c) State, giving a reason, why Tisam's model of this game is not suitable to describe Nav playing the game for all values of X

(1)

Question 6

The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers $(a < b < c)$
- all the probabilities are greater than zero

(a) Find

(i) the value of a

(ii) the value of b

(iii) the value of c

Show your working clearly.

(5)

The independent random variables X_1 and X_2 each have the same distribution as X

(b) Find $P(X_1 = X_2)$

(2)

Question 7

The discrete random variable D has the following probability distribution

d	10	20	30	40	50
$P(D = d)$	$\frac{k}{10}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{40}$	$\frac{k}{50}$

where k is a constant.

(a) Show that the value of k is $\frac{600}{137}$

(2)

The random variables D_1 and D_2 are independent and each have the same distribution as D.

(b) Find $P(D_1 + D_2 = 80)$
Give your answer to 3 significant figures.

(3)

A single observation of D is made.

The value obtained, d, is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral Q

(c) Find the exact probability that the smallest angle of Q is more than 50°

(5)