
A LEVEL MATHEMATICS

SEQUENCES AND SERIES

NAME _____

Answers based on calculator technology are not acceptable

Question 1

A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = 3(u_n)^2 - 1$$
$$u_1 = k$$

where k is a real constant.

(a) Show that

$$u_3 = 27k^4 - 18k^2 + 2 \quad (3)$$

Given that

$$u_3 - 5u_2 = 11$$

(b) find the possible values of k .

(3)

Question 2

An arithmetic series has first term a and common difference d , where a and d are constants.

Given that the sum of the first 65 terms of the series is 4940

(a) show that

$$a + 32d = 76 \quad (2)$$

Given also that the 49th term of the series is 64

(b) write down another equation connecting a and d . (1)

(c) Hence find the value of a and the value of d . (2)

Given that S_n represents the sum of the first n terms of the series,

(d) find the largest possible positive value of S_n
(Solutions relying entirely on calculator technology are not acceptable.) (3)

Question 3

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

The 3rd, 4th and 5th terms of a geometric sequence are $\sin \theta$, $\sqrt{2} \cos \theta$ and $\sqrt{3} \cot \theta$ respectively, where θ is a constant, $0 < \theta < \frac{\pi}{2}$

(a) Find the exact value of the common ratio of this sequence.

(4)

(b) Hence find, in simplest form, the exact value of the sum of the first 3 terms of the sequence.

(3)

Question 4

A sequence of terms $a_1, a_2, a_3, \dots$ is defined by

$$\begin{aligned}a_1 &= 4 \\a_{n+1} &= ka_n + 3\end{aligned}$$

where k is a constant.

Given that

- $\sum_{n=1}^3 a_n = 12$
- all terms of the sequence are different

find the value of k

(4)

Question 5

In this question you must show all stages of your working.

Given that the first three terms of a geometric sequence are

$$8 \sin \theta \qquad 3 \sin 2\theta \qquad 2 + 2 \cos 2\theta$$

$$\text{and } 0 < \theta < \frac{\pi}{2}$$

(a) show that

$$\sin \theta = \frac{8}{9} \qquad (4)$$

Hence, or otherwise

(b) prove that S_{∞} exists (2)

(c) find the value of $S_{\infty} - S_{10}$ giving your answer to 2 significant figures. (3)

Question 6

The first three terms of an arithmetic sequence are

$$6k, 10 \text{ and } 2k$$

where k is a constant.

(a) Find the value of k .

(2)

(b) Hence find the value of the sum of the first 50 terms of this sequence.

(3)

Question 7

A sequence $u_1, u_2, u_3 \dots$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7 \cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

Question 8

The first three terms of a geometric sequence are

$$3k + 4 \quad 12 - 3k \quad k + 16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0 \quad (2)$$

Given that the sequence converges,

(b) (i) find the value of k , giving a reason for your answer,

(ii) find the value of S_{∞} (5)