A LEVEL MATHEMATICS

Trigonometry

NAME_____

Instructions

- **Answer all questions**
- **Show complete working out,**
- **Write neatly and legibly**

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$$

Small angle approximations

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan \theta \approx \theta$$

where θ is measured in radians

Question 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$(\cos\theta + \sin\theta)(\operatorname{cosec}\theta - \sec\theta) \equiv k\cot 2\theta \qquad \theta \neq (90n)^\circ \quad n \in \mathbb{Z}$$

where k is an integer to be found.

(3)

(b) Hence solve, for $-90° < x < 90°$, the equation

$$5(\cos x + \sin x)(\operatorname{cosec} x - \sec x) = 4\operatorname{cosec}^2 2x$$

Give each answer, in degrees, to one decimal place.

(5)

Question 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Express $4\cos x - 13\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

The demand for electricity, D gigawatts (GW), in the UK on a particular day, is modelled by the equation

$$D = 30 + 4\cos\left(\frac{\pi t}{12} + 0.2\right) - 13\sin\left(\frac{\pi t}{12} + 0.2\right) \qquad 0 < t < 24$$

where t is the number of hours after midnight.

Use the equation of the model and the answer to part (a), to answer parts (b) and (c).

(b) (i) Deduce the minimum demand for electricity during this day.

(ii) Find the time of the day when this minimum demand occurs.

(4)

At 3 pm on this day, the demand for electricity is increasing at a rate of k GW per hour.

(c) Find the value of the constant k, giving the answer to 3 significant figures.

(3)

(d) Give a reason why it would not be sensible to use the same equation to model the demand for electricity in the UK on every day of the year.

(1)

Question 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that

$$\sin(x + 30°) + \sqrt{3}\cos(x + 30°) \equiv 2\cos x$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 180°$

$$\sin(\theta + 30°) + \sqrt{3}\cos(\theta + 30°) = 3\sin 2\theta$$

giving your answers to one decimal place, where appropriate.

(4)

Question 4

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1} \equiv 2\tan\theta\sec\theta \qquad \theta \neq (90n)^\circ,\ n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < x < 90°$, the equation

$$\frac{1}{\operatorname{cosec} 2x - 1} + \frac{1}{\operatorname{cosec} 2x + 1} = \cot 2x \sec 2x$$

Give each answer, in degrees, to one decimal place.

(4)

Question 5

. (a) Express $140\cos\theta - 480\sin\theta$ in the form $K\cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90°$

State the value of K and give the value of α, in degrees, to 2 decimal places.

(3)

A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, R, is modelled by the equation

$$R = A + 140\cos(30t)° - 480\sin(30t)°$$

where t months is the time after the start of the year and A is a constant.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(b) (i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.

(2)

The actual number of rabbits in the wood is at its minimum value in the middle of April.

(c) Use this information to comment on the model for the number of rabbits.

(2)

The number of foxes, F, in the wood during the same year is modelled by the equation

$$F = 100 + 70\sin(30t + 70)°$$

The number of foxes is at its minimum value after T months.

(d) Find, according to the models, the number of **rabbits** in the wood at time T months.

(4)

Question 6

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.

(3)

(b) Hence, solve for $360° \leqslant x \leqslant 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

(4)

Question 7

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)