AS LEVEL MATHEMATICS

GRAPHS AND TRANSFORMATIONS TEST

NAME Marked Solutions

Instructions

- **Answer all questions**
- **Show complete working out,**
- **For any graphs clearly shown the points where graphs crosses the coordinate axis and/or the asymptotes**
- **Write neatly and legibly**

Question 1

The point $P(-4, 5)$ lies on the continuous curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$
Find the coordinates of the point to which P is mapped when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = 4\mathrm{f}(2x)$

$(-2, 20)$

(2)

(b) $y = \mathrm{f}(x + 3) - 2$

$(-7, 3)$

(2)

Question 2

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

$f(x) = 2[x^2 + 2x] + 9$

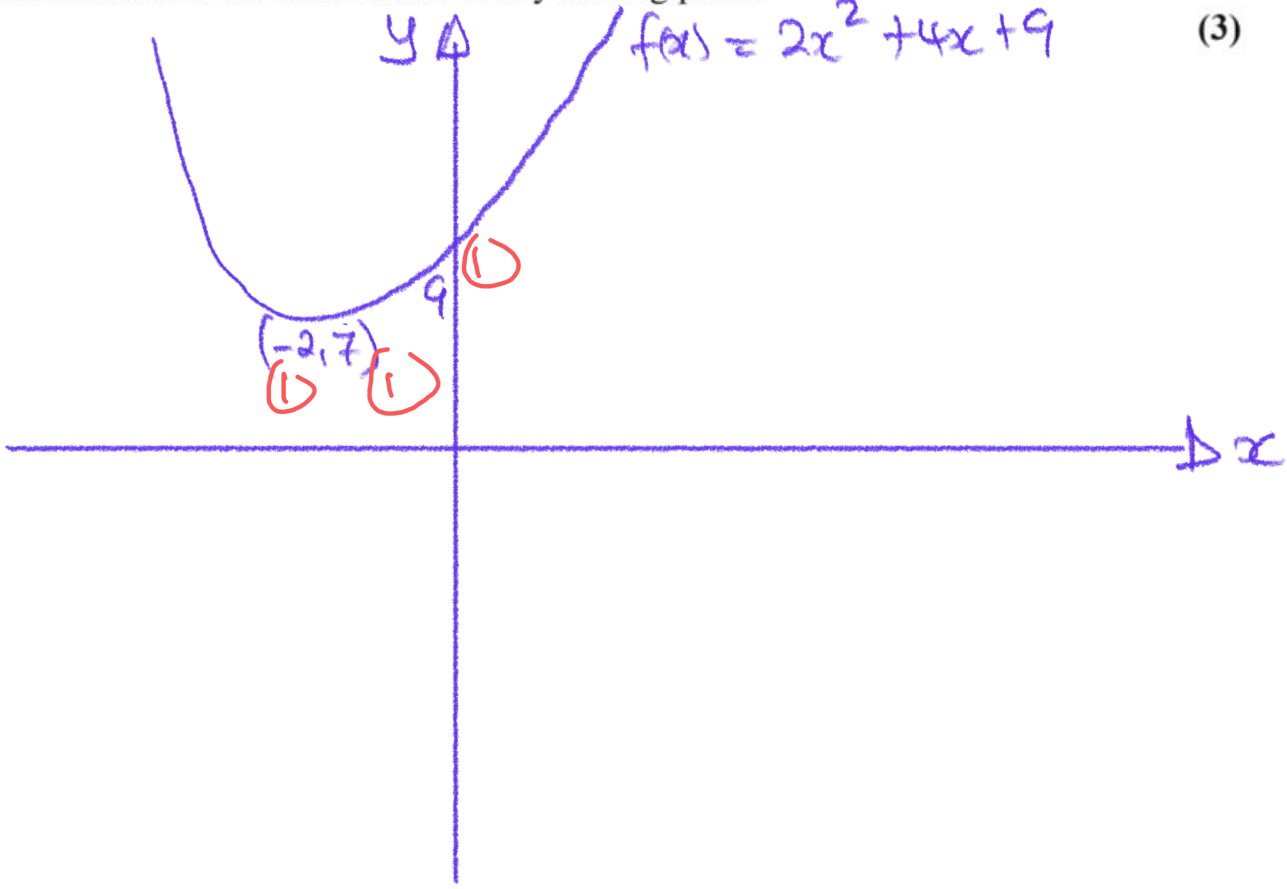
$= 2[(x+1)^2 - 1] + 9$ (1)

$= 2(x+1)^2 - 2 + 9$ (1)

$= 2(x+1)^2 + 7$ (1)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

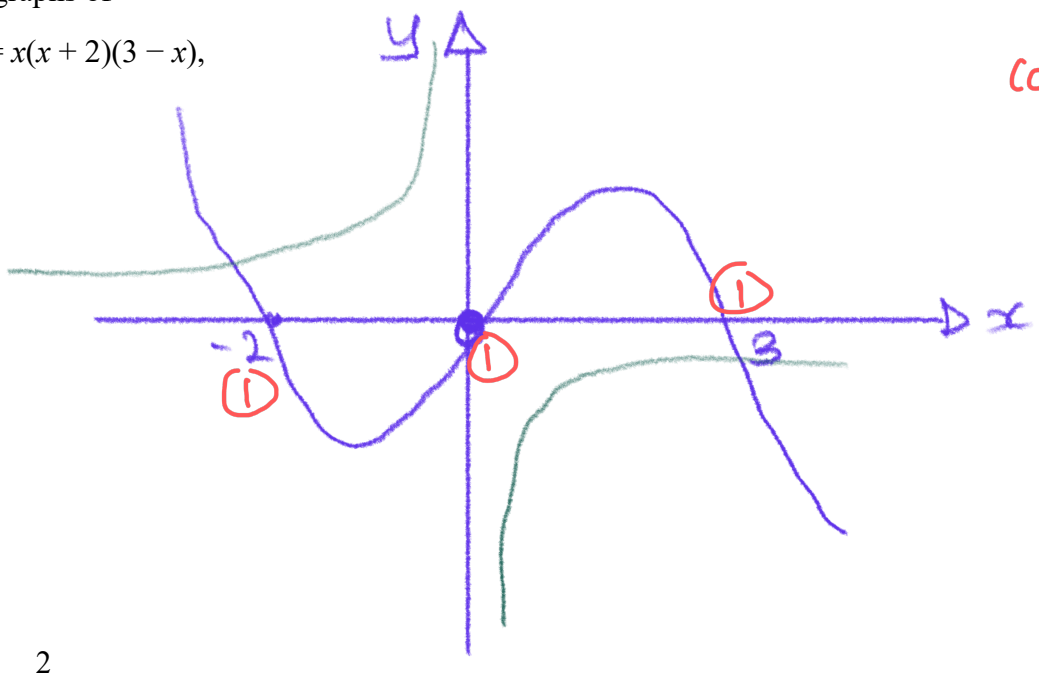


(1) (1) (1)

Question 3

(a) Sketch the graphs of

(i) $y = x(x+2)(3-x)$,

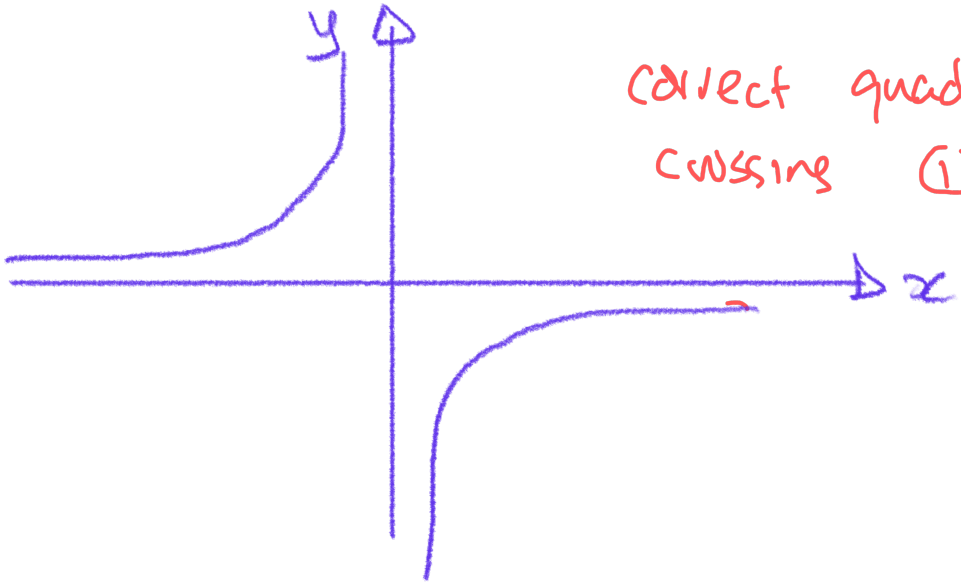


correct shape (1)

(ii) $y = -\frac{2}{x}$.

showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)



correct quadrant (1)
crossing (1)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0.$$

2 real roots, as these are only a points of intersection between the graphs.

(2)

Question 4

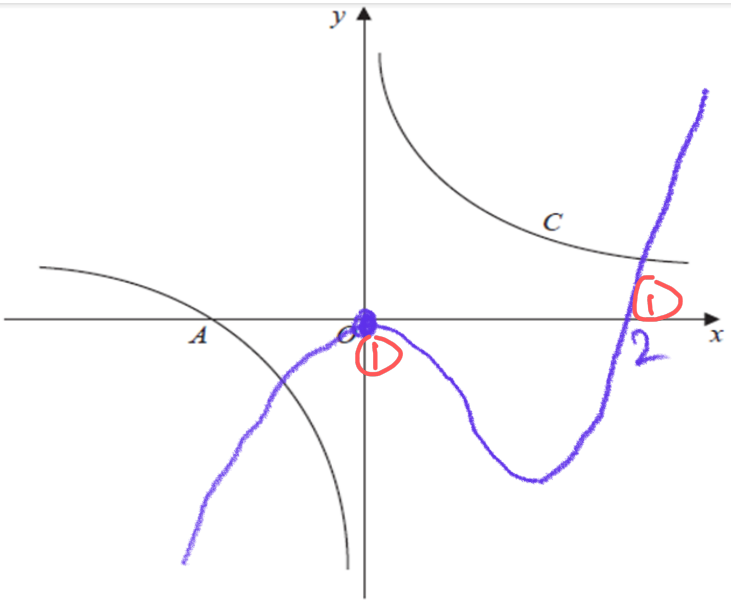


Figure 4

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{x} + 1, \quad x \neq 0.$$

The curve C crosses the x-axis at the point A.

(a) State the x-coordinate of the point A.

$0 = \frac{1}{x} + 1$

$0 = 1 + x \quad \therefore \quad x = -1$ (1)

(1)

The curve D has equation $y = x^2(x - 2)$, for all real values of x.

(b) On a copy of Figure 1, sketch a graph of curve D. Show the coordinates of each point where the curve D crosses the coordinate axes.

Shape (1)

(3)

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x - 2) = \frac{1}{x} + 1$$

(1)

2 solutions as curve crosses twice (1)

Question 5

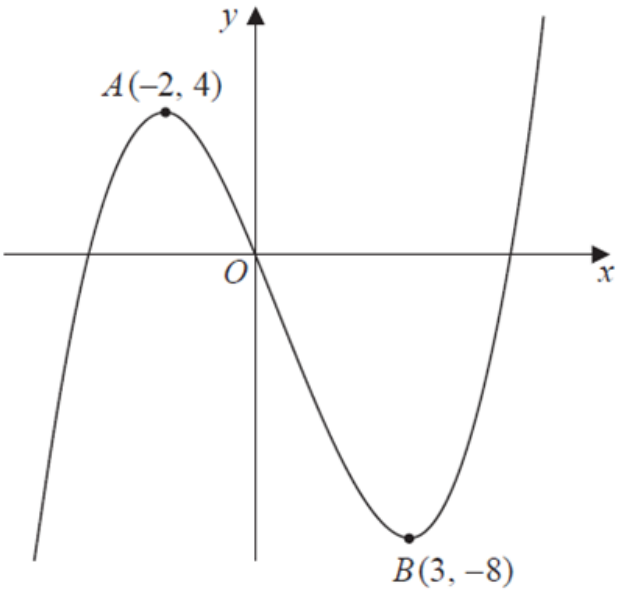


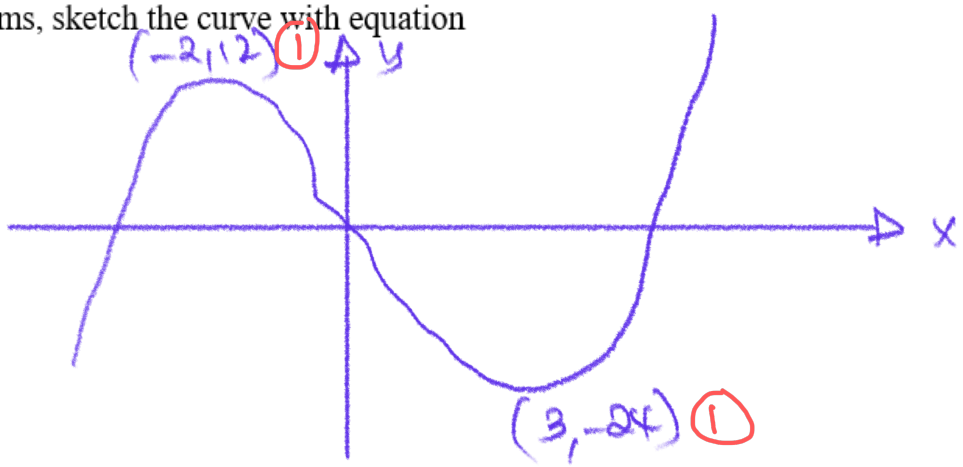
Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$. The curve has a maximum point A at $(-2, 4)$ and a minimum point B at $(3, -8)$ and passes through the origin O.

On separate diagrams, sketch the curve with equation

(a) $y = 3f(x)$,

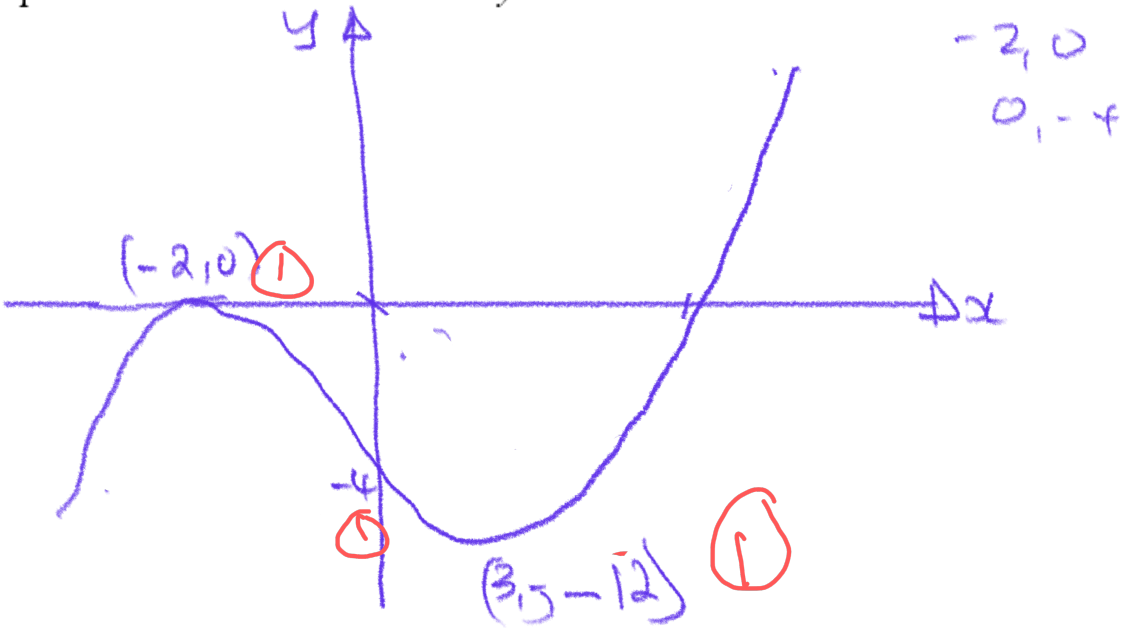
(2)



(b) $y = f(x) - 4$.

(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.



Question 6

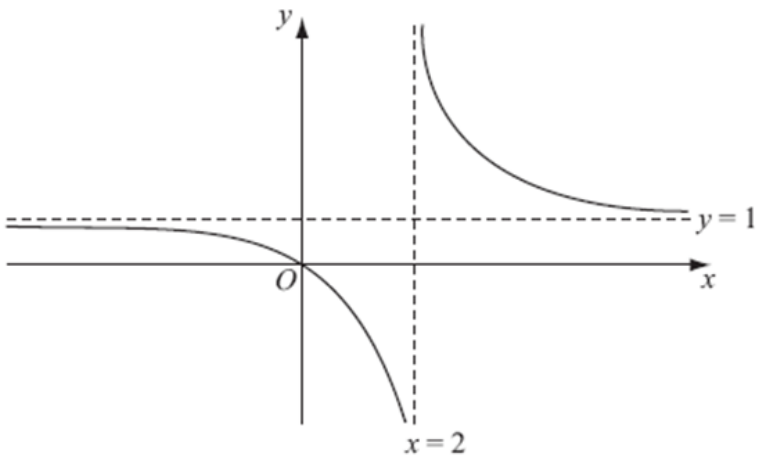


Figure 3

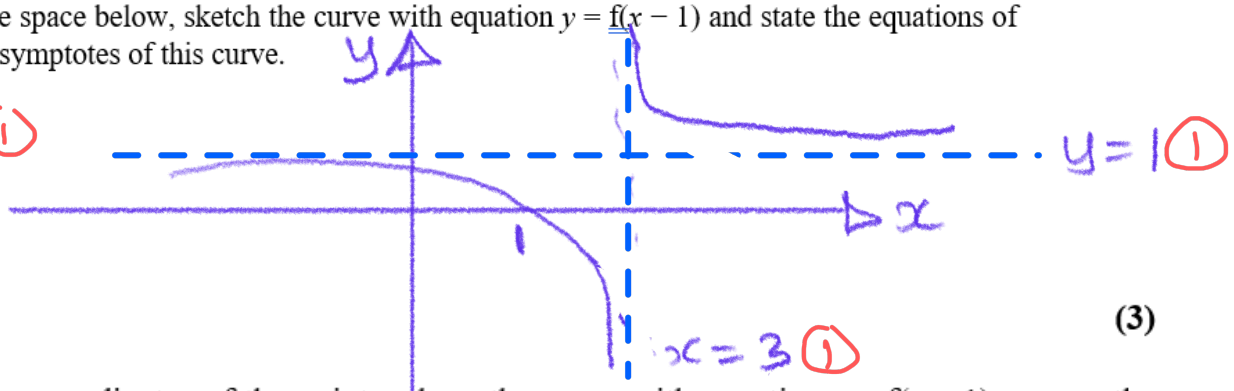
$y = \frac{0}{-2}$

Figure 3 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \frac{x}{x-2}, \quad x \neq 2.$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = f(x - 1)$ and state the equations of the asymptotes of this curve.



correct shape (1)

(3)

(b) Find the coordinates of the points where the curve with equation $y = f(x - 1)$ crosses the coordinate axes.

(4)

$y = \frac{x-1}{x-3}$ (1)

$y = \frac{1}{3}$ (1)

$\therefore (0, \frac{1}{3})$ (1) and $(1, 0)$ (1)

Question 7

(a) Factorise completely $9x - 4x^3$.

$x(9 - 4x^2$ (1)

$x(3 - 2x)(3 + 2x)$ (1)

(1)

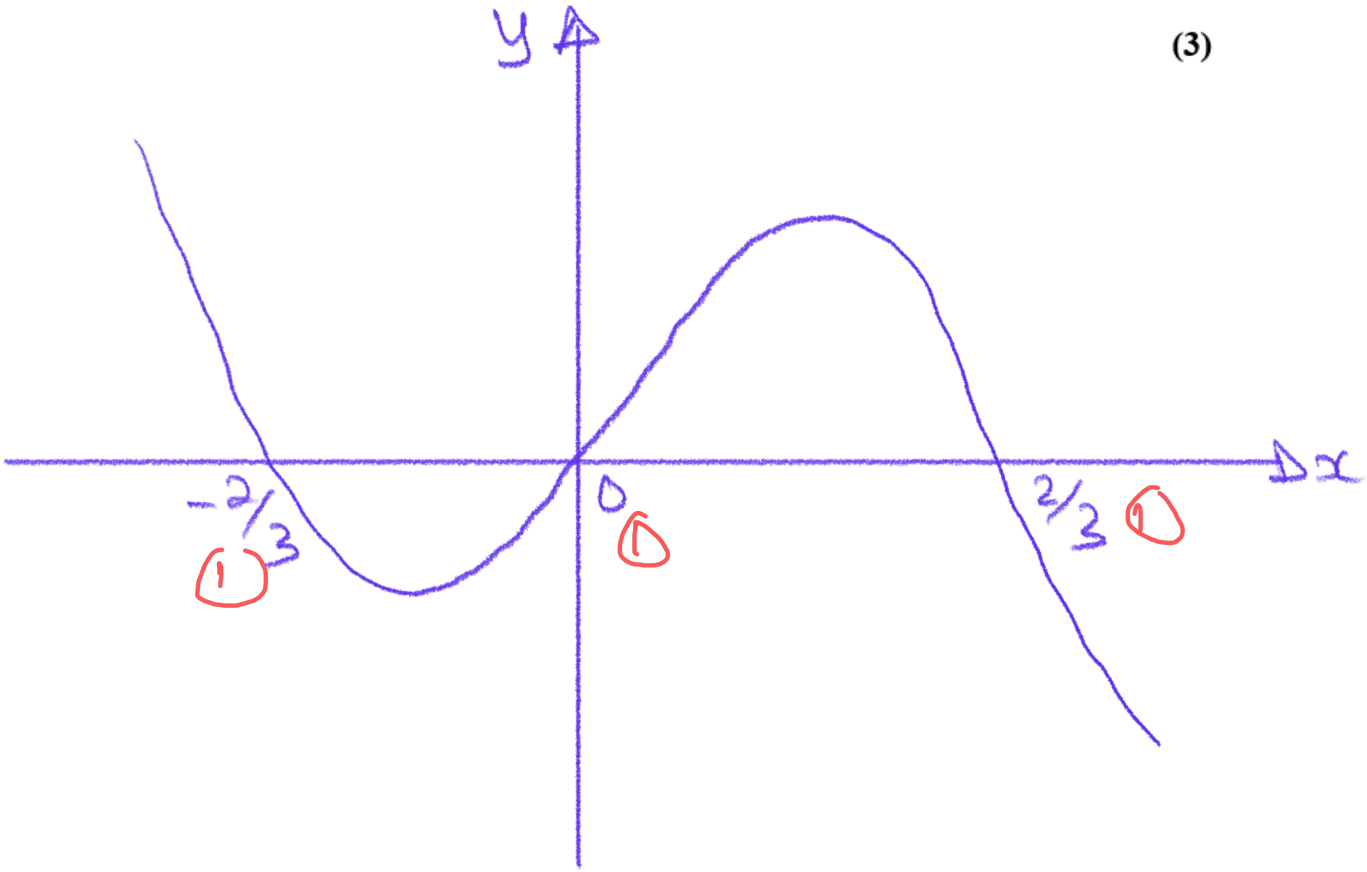
(3)

(b) Sketch the curve C with equation

$$y = 9x - 4x^3.$$

Show on your sketch the coordinates at which the curve meets the x-axis.

(3)



Question 8

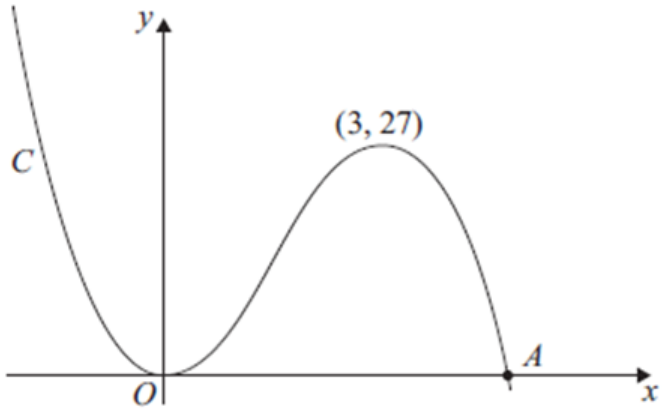


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = x^2(9 - 2x).$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

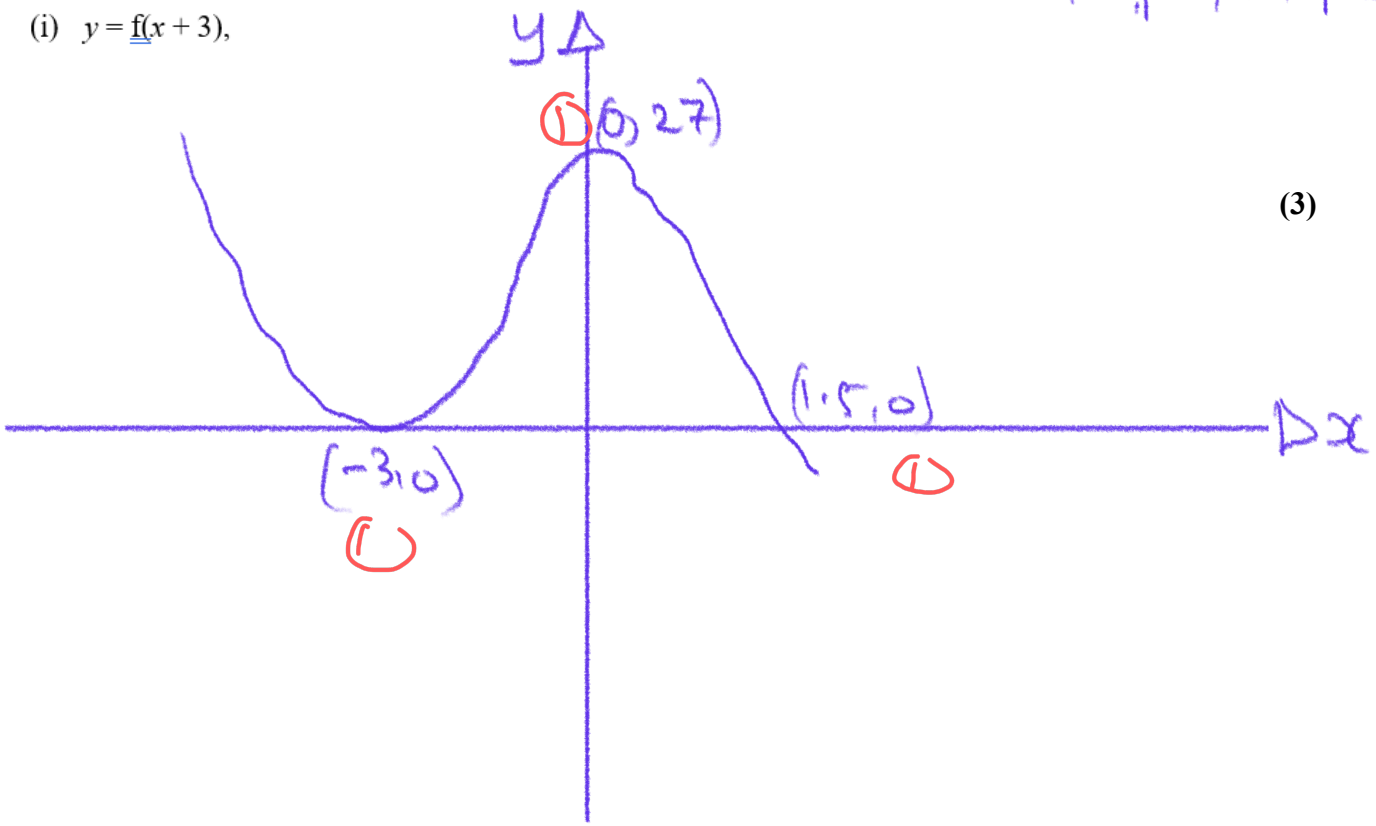
(a) Write down the coordinates of the point A. $(4\frac{1}{2}, 0)$

(1)

(b) On separate diagrams sketch the curve with equations below. On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

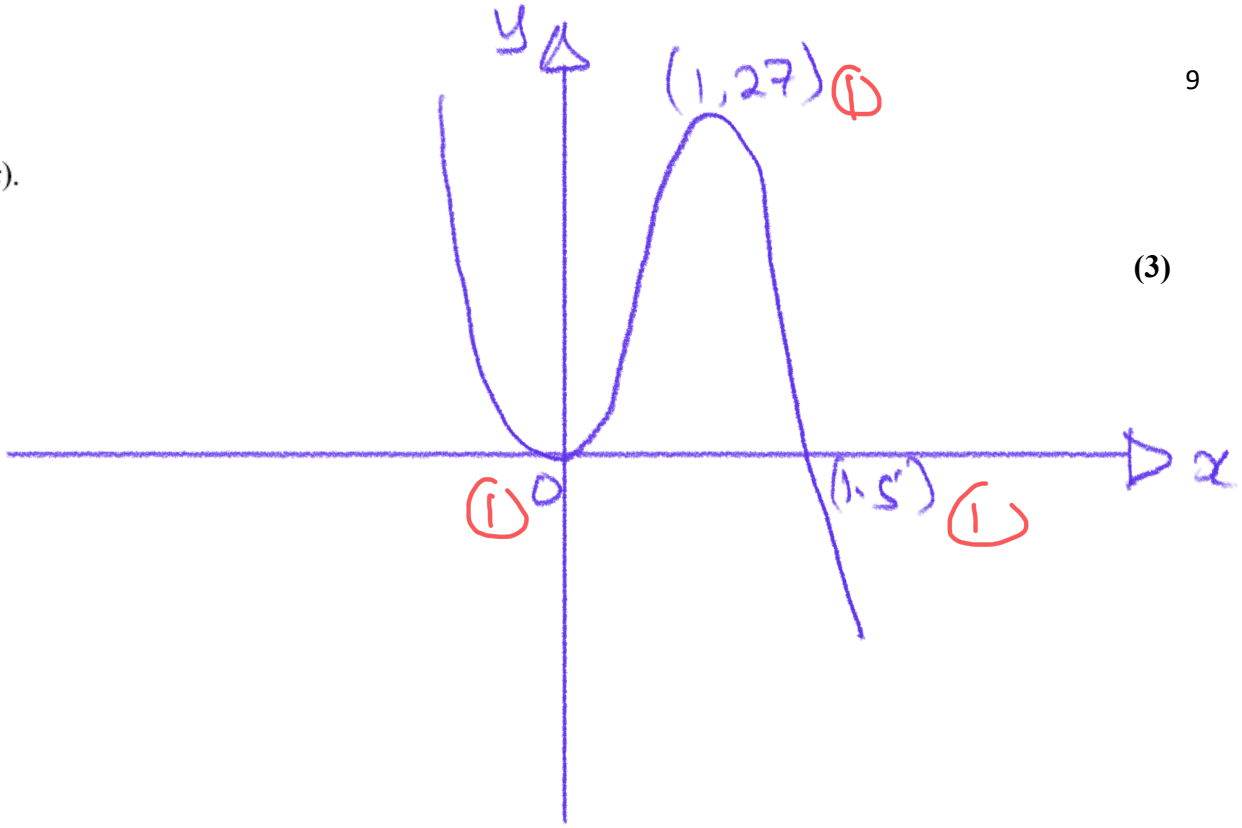
$-3, 0$ | $0, 27$ | $1.5, 0$

(i) $y = f(x + 3)$,



(3)

(ii) $y = f(3x)$.



(3)

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

$k = -17$

(2)

Total = 50 marks